

逢甲大學109學年度碩士班考試入學試題

編號：06 科目代碼：105

科目	微積分	適用 系所	統計學系統計與精算碩士班應 用統計暨計量財務組、精算組	時間	90分鐘
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※請務必在答案卷作答區內作答。

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1. (15%) Evaluate the following limits

(a). (5%) $\lim_{n \rightarrow \infty} \frac{\sqrt[3]{x^3+x+4}}{4+x}$

(b). (5%) $\lim_{n \rightarrow 1} \left(\frac{2}{x^2-1} - \frac{1}{x-1} \right)$

(c). (5%) $\lim_{x \rightarrow 64} \frac{\sqrt[6]{x}-2}{\sqrt[3]{x}+\sqrt{x}-12}$

2. (15%) Find the first derivative of the following function.

(a). (5%) $f(x) = x^x$

(b). (5%) $f(x) = x^{x^x}$

(c). (5%) $f(x) = x^{a^a} + a^{x^a} + a^{a^x}$

3. (10%) Find $\frac{dy}{dx}$

(a). (5%) $\frac{x+y}{x-y} = xy$

(b). (5%) $x = \sin xy$

4. (10%) Determine whether the series $\sum_{n=1}^{\infty} \frac{1}{3^n+n}$ is convergent or divergent. Explain.

5. (24%) Compute the following integrals.

(a) (4%) $\int_{-2}^2 \frac{x^3}{1+x^2+x^4} dx$

(b) (4%) $\int_0^{\infty} x^5 e^{-x} dx$

(c) (4%) $\int_0^1 x^4(1-x)^4 dx$

(d) (4%) $\int_0^{\infty} \lambda x^2 e^{-\lambda x} dx$, where λ is a constant.

(e) (4%) $\int_1^e \frac{\ln x}{x} dx$

(f) (4%) $\int_a^b \frac{e^{tx}}{b-a} dx$, where t , a , and b are constants.

6. (8%) Find the inflection point(s) of

$$g(x) = \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right), -\infty < x, \mu < \infty, \sigma > 0.$$

7. (8%) A nonnegative random variable has a hazard rate function of

$$h(x) = A + e^{2x}, x \geq 0.$$

We are also given $S(0.2)=0.8$. Determine the value of A . Note that

$$S(b) = \exp\left(-\int_0^b h(x) dx\right)$$

8. (10%) Evaluate the double integral.

(a) (5%) $\int_1^4 \int_0^{\sqrt{y}} e^{x/\sqrt{y}} dx dy$

(b) (5%) $\iint_D \frac{\sin x}{x} dA$, where D is the triangle $\{(x, y) : 0 \leq y \leq x, 0 \leq x \leq \pi\}$