逢甲大學109學年度碩士班考試入學試題

編號：08　科目代碼：202

科目	統計學	適用系所	經濟學系	時間	90分鐘

※請務必在答案卷作答區內作答。　

一、選擇題（每題4分），共48分

1. Suppose the GDP growth rates of Taiwan calculated on a year-to-year basis over the past three years are r_1, r_2, and r_3, respectively. Assume the *average* annual GDP growth rate of Taiwan over past three years is r. What is the *exact* relation of r_1, r_2, r_3, and r?
   A) $3r = r_1 + r_2 + r_3$
   B) $r^3 = r_1 * r_2 * r_3$.
   C) $3\log(1+r) = \log(1+r_1) + \log(1+r_2) + \log(1+r_3)$.
   D) $(\log(1+r))^3 = \log(1+r_1) * \log(1+r_2) * \log(1+r_3)$.

2. Consider the following four statements:
   (1) Events A and B are mutually exclusive, then they are independent.
   (2) Events A and B are independent, then they are mutually exclusive.
   (3) Random variables X and Y are uncorrelated, then they are independent.
   (4) Random variables X and Y are independent, then they are uncorrelated.
   How many statements are true?
   A) One.　B) Two.　C) Three.　D) Four.

3. Suppose X and Y are two random variables. Consider the following four statements:
   (1) If $\mathrm{E}[X+Y] = \mathrm{E}[X] + \mathrm{E}[Y]$, then X and Y are uncorrelated.
   (2) If $\mathrm{E}[XY] = \mathrm{E}[X]\mathrm{E}[Y]$, then X and Y are uncorrelated.
   (3) If X and Y are uncorrelated, then $\mathrm{Var}[X+Y] = \mathrm{Var}[X] + \mathrm{Var}[Y]$.
   (4) If X and Y are uncorrelated, then $\mathrm{Var}[XY] = \mathrm{Var}[X]\mathrm{Var}[Y]$.
   How many statements are true?
   A) One.　B) Two.　C) Three.　D) Four.

4. The average original grade of an economic course is 55, and the variation of the original scores is 120. The professor intends to make the linear adjustments, that is, the original score is X and the adjusted score Y follow the relation $Y = aX + b$. She also intends to adjust the original score 45 to 60 and adjust the highest score 73 to 95. What are the mean and the variation of adjusted scores?
   A) 70; 175　B) 72; 182.5　C) 72.5; 187.5　D) 74; 192.5

5. A statistics professor classified his students according to their grade point average (GPA, random variable Y) and their gender (random variable X). The accompanying table shows the proportion of students falling into various categories.

	Under 2.0 ($Y=0$)	2.0-3.0 ($Y=1$)	3.0-4.0 ($Y=2$)
Male ($X=0$)	0.10	0.20	0.10
Female ($X=1$)	0.10	0.35	0.15

   One student is selected at random. What is the approximated value of the conditional variance of Y conditional on $X=1$, $\mathrm{Var}[Y|X=1]$?
   A) 0.36　B) 0.41　C) 0.46　D) 0.51

6. Following the previous question, what is the approximated value of the correlation coefficient between X and Y, $\mathrm{Cov}(X, Y)$?
   A) -0.06 B) -0.02 C) 0 D) 0.02

7. In a survey of 2500 likely voters, 1997 responded that they would vote for the incumbent and 503 responded that they would vote for the challenger. Let p denote the fraction of all likely voters who preferred the incumbent at the time of the survey, and let $\hat{p}$ denote the fraction of survey respondents who preferred the incumbent. Which of the following values is closest to the stand error of $\hat{p}$?
   A) 0.008 B) 0.01 C) 0.4 D) 0.5

8. Suppose X_1, X_2, and X_3 are *i.i.d.* drawn from Bernoulli distribution $\mathrm{Bern}(0.7)$. Suppose $Y = X_1 + X_2X_3$. Consider the following four statements:
   (1) $\mathrm{E}(Y) = 1.19$.
   (2) $\mathrm{Var}(Y) = 0.4599$.
   (3) $\mathrm{Prob}(Y > 0) = 1 - 0.3^3$
   (4) $\mathrm{Prob}(Y > 0) = \mathrm{Prob}(Y > 0.5)$
   How many statements are true?
   A) One. B) Two. C) Three. D) Four.

9. Suppose both random variables X and Y follow the normal distribution. Consider the following four statements:
   A) $\mathrm{Prob}(X = 0) = 0$.
   B) $\mathrm{Prob}(X > \mathrm{E}(X)) = 0.5$.
   C) If X and Y are uncorrelated, then they are independent.
   D) $X + Y$ also follows a normal distribution.
   How many statements are true?
   A) One. B) Two. C) Three. D) Four.

10. Which one of the following statements about central limit theorem is true?
    A) It states that under general conditions, when the sample size is large, sample average would be close to the population mean with high probability.
    B) The theorem is applied on any sample size greater than 30.
    C) It states that under general conditions, when the sample size is large, the sampling distribution of the standardized sampling average is approximated standard normal.
    D) None of above.

11. Considering the hypothesis, H_0: $\mu \geq \mu_0$ vs. H_1: $\mu < \mu_0$, when the sample size n becomes larger and the other conditions are invariable. Find the correct statement as follows.
    A) The probabilities of type I error and type II error both become larger.
    B) The probabilities of type I error and type II error both become smaller.
    C) The probability of type I error becomes larger, but the the probability of type II error becomes smaller.
    D) The probability of type I error becomes smaller, but the the probability of type II error becomes larger.

12. Under Normal distribution, the $(1 - \alpha)$ 100% confidence interval of population mean μ is $\overline{\mathrm{X}} \pm 1.753 \cdot \mathrm{s}/4$, find the confidence level $(1 - \alpha)$ 100%.
    A) 80%
    B) 90%
    C) 95%
    D) 99%

二、計算題，共52分

1. Suppose a disease is present in 3% of population. A diagnostic test shows 5% false positives and 2% false negatives. That is, for a patient not having the disease, the test shows positive (+) with probability 0.05 and negative (-) with probability 0.95; for a patient having the disease, the test shows positive with probability 0.98 and negative with probability 0.02.
   (a) If a patient's test shows (-), what is the probability that he does not have the disease? (4%)
   (b) If a patient's test shows (+), what is the probability that he indeed has the disease? (4%)
   (c) Consider a patient's test shows (+). He is rediagnosed and the second test still shows (+). Suppose the two tests are independent and they have the same false positive probability and false negative probability. What is the probability that he indeed has the disease? (6%)

2. The coronavirus has been in the news and is continuing to be a concern to all of us. Consider the body temperature averaged 40° with a standard deviation of 1.5° in a random sample of 13 patients from the hospital. We are interested in determining if the mean of body temperature is significantly different from 39.5°.
   (a) Conduct a hypothesis test with significant level $\alpha = 0.05$ to test if the average body temperature equals to 39.5° or not. (5%)
   (b) Compute the p-value. (5%)
   (c) Using the p-value method at the 95% confidence interval, would you draw conclusions from the confidence interval results? (4%)

3. In order to minimize the risk of infection, the universities are currently agreeing to postpone the classes. The postponed days can be estimated with a linear relationship involving the number of infected persons from coronavirus. In the following table, the first row lists the number of infected people of six universities, and the second row lists the postponed days of the universities corresponding to the first row.

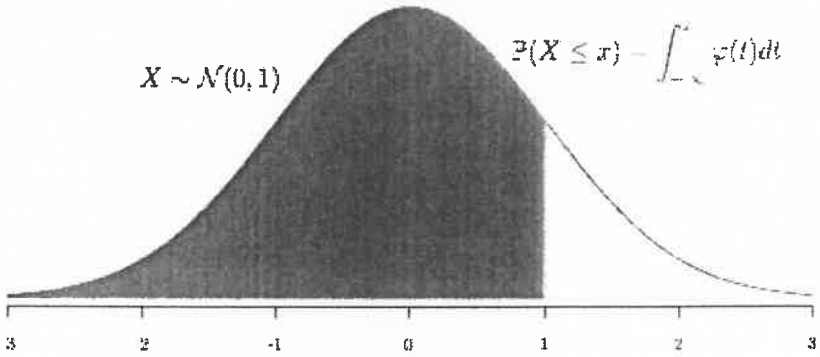
Infected people, X	**15**	**8**	**13**	**21**	**14**	**28**
Postponed days, Y	**21**	**14**	**20**	**28**	**23**	**35**

The summary statistics are:

$$\sum_{i=1}^{6} X_i = 99;\ \sum_{i=1}^{6} Y_i = 141;\ \sum_{i=1}^{6} X_i^2 = 1{,}879;\ \sum_{i=1}^{6} Y_i^2 = 3{,}575$$

   (a) State the required assumption for conduction linear regression analysis. (4%)
   (b) Test for a linear relationship between infected people and postponed days. The null hypothesis is H_0: $\rho_{xy} = 0$ while the alternative hypothesis is H_1: $\rho_{xy} \neq 0$. (5%)
   (c) What is a 95% confidence interval for the change in postponed days for one unit change in infected people? (5%)
   (d) Calculate the coefficient of determination, R^2, and interpret it. (4%)
   (e) Find a 95% confidence interval for the value of the postponed days that corresponds to the number of infected people being 11. (4%)

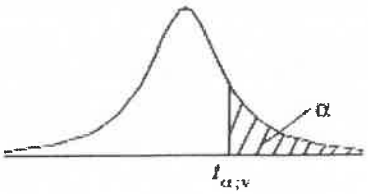
Z表：



	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

T表：

Table of the Student's *t*-distribution



The table gives the values of $t_{\alpha,\nu}$ where
$\Pr(T_\nu > t_{\alpha,\nu}) = \alpha$, with ν degrees of freedom

ν \ α	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
1	3.078	6.314	12.076	31.821	63.657	318.310	636.620
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291