1. (30%)　John wants to choose his password for his new email account. How many different passwords are possible if a password is required to consist of

(a) 6 digits (0 to 9)

(b) 6 **distinct** digits (0 to 9) , for example, 521467 or 216098?

(c) 6 **strictly increasing** digits (0 to 9), for example, 014679 or 134689?

(d) 6 **increasing** digits (0 to 9), for example, 011467 or 111344?

(e) 6 digits (0 to 9) of which the **sum** is 9, for example, 321103 or 110044?

(f) 6 digits (0 to 9) of which the **sum** is 20, for example, 326702 or 117344?

2. (10%) Show that $[(q \rightarrow p) \wedge p] \rightarrow (p \vee q)$ is always true.

3. (9%) Determine which of the following arguments are valid and which are invalid. Provide an explanation for each answer.

a) All students go to school.　　　(b) All citizens pay their taxes.

John is a student　　　　　　　　　Tom pays his taxes.

Therefore John goes to school.　　　Therefore Tom is a citizen.

(c) All people who are concerned about the environment recycle their plastic containers.

Margarita is not concerned about the environment.

Therefore Margarita does not recycle her plastic containers.

4. (10%) Use the Principle of Mathematical Induction to prove that

$$(1 - \frac{1}{4}) \times (1 - \frac{1}{9}) \times (1 - \frac{1}{16}) \times \cdots \times (1 - \frac{1}{n^2}) = \frac{n+1}{2n}, \text{ for } n \geq 2.$$

5. (10%) If $n \in Z^+$ with $n \geq 4$, show that the number of assignments to assign n people into n-2 groups without group labeling is $\binom{n}{3} + 3\binom{n}{4}$.

6. (10%) S = {1, 2, 3, 4, 5, 6, 7, 8, 9}. If six elements are selected from S, show that two of them must have the sum 10.

7. (8%) Consider the students of a class. Let R is the relation that $x R y$ if x and y live in a same hometown. Determine whether R is reflexive, symmetric, antisymmetric, or transitive.

8. (13%) S = {1, 2, 3, 4, 5, 6, 7}. x, y∈S. Let x R y if x| y.

(a) Show the Hasse diagram of R.

(b) What are the (i) maximal, (ii) minimal, (iii) greatest, and (iv) least elements of the poset (S, R)?

(c) B={2, 3}. What are the (i) lower and (ii) upper bounds of B?