國立高雄大學 109 學年度研究所碩士班招生考試試題

系所：應用數學系

科目：微積分　　身份別：一般生應用數學組、在職生應用數學組　　是否使用計算機：否

考試時間：100 分鐘

本科原始成績：100 分

1. (10 %) Determine whether the statement is True or False.
   (a) If $f(x) < g(x)$ for all $x \neq a$, then $\lim_{x\to a} f(x) < \lim_{x\to a} g(x)$.
   (b) The graph of every cubic polynomial has precisely one point of inflection.
   (c) It is possible to find a power series whose interval of convergence is $[0, \infty)$.
   (d) The polar equations $r = \sin 2\theta$, $r = -\sin 2\theta$, and $r = \sin(-2\theta)$ all have the same graph.
   (e) Two different level curves of the graph of $z = f(x, y)$ can intersect.

2. (5 %) Find the derivative of $g(x) = \displaystyle\int_{\tan x}^{x^2} \frac{1}{\sqrt{2+t^4}}\, dt$

3. (5 %) Determine whether $\displaystyle\sum_{n=1}^{\infty} \left(2\sqrt[n]{n} + 1\right)^n$ converges or diverges. Identify the test used.

4. (10 %) Let $f(x) = \dfrac{x+6}{x-2}$, $x > 2$. Verify that f has an inverse function and find $(f^{-1})'(3)$.

5. (10 %) Evaluate $\lim_{x\to 0^+} (e^x + x)^{2/x}$

6. (10 %) Find the unit tangent vector and the principal unit normal vector to the curve $\mathbf{r}(t) = t\,\mathbf{i} + \frac{1}{2}t^2\,\mathbf{j}$ at $t = 2$.

7. (10 %) Show that $f_x(0,0)$ and $f_y(0,0)$ both exist but that f is not differentiable at $(0,0)$.
$$f(x,y) = \begin{cases} \dfrac{3x^2y}{x^4+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

8. Find or evaluate the integral.
   (a) (10 %) $\displaystyle\int \frac{x}{\sqrt{4x - x^2}}\, dx$
   (b) (10 %) $\displaystyle\int_0^1 x \ln x\, dx$
   (c) (10 %) $\displaystyle\int_0^1 \int_{y/2}^{1/2} e^{-x^2}\, dx dy$
   (d) (10 %) $\displaystyle\int_{-1}^1 \int_0^{\sqrt{1-x^2}} \cos(x^2 + y^2)\, dy\, dx$

背面尚有試題