

國立中山大學 109 學年度 碩士暨碩士專班招生考試試題

科目名稱：機率【通訊所碩士班甲組】

— 作答注意事項 —

考試時間：100 分鐘

- 考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
- 答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，不得另攜帶紙張，請衡酌作答。
- 答案卡請以 2B 鉛筆劃記，不可使用修正液（帶）塗改，未使用 2B 鉛筆、劃記太輕或污損致光學閱讀機無法辨識答案者，其後果由考生自行負擔。
- 答案卷（卡）應保持清潔完整，不得折疊、破壞或塗改應考證號碼及條碼，亦不得書寫考生姓名、應考證號碼或與答案無關之任何文字或符號。
- 可否使用計算機請依試題資訊內標註為準，如「可以」使用，廠牌、功能不拘，唯不得攜帶具有通訊、記憶或收發等功能或其他有礙試場安寧、考試公平之各類器材、物品（如鬧鈴、行動電話、電子字典等）入場。
- 試題及答案卷（卡）請務必繳回，未繳回者該科成績以零分計算。
- 試題採雙面列印，考生應注意試題頁數確實作答。
- 違規者依本校招生考試試場規則及違規處理辦法處理。

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題號：437005

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（混合題）

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一、選擇題(單選，計分方式:不倒扣，答對得該題全部分數，答錯及未作答得零分)

1. (5%) Let E^c denote the complement of an event E . Which of the following pairs of events, A and B , can be disjoint?

- (A) $\Pr(A) = \frac{1}{3}$ and $\Pr(B^c) = \frac{1}{4}$
- (B) $\Pr(A) = \frac{1}{4}$ and $\Pr(B^c) = \frac{1}{2}$
- (C) $\Pr(A) = \frac{1}{2}$ and $\Pr(B) = \frac{2}{3}$
- (D) $\Pr(A) = \frac{1}{2}$ and $\Pr(B) = \frac{4}{5}$
- (E) None of these

2. (5%) The random variable X has the probability density function

$$f_X(x) = \begin{cases} cx^{-2} & \text{if } 1 \leq x \leq 2; \\ 0 & \text{otherwise.} \end{cases}$$

Let A be the event $\{X > 3/2\}$ and $Y = X^2$. What is the conditional variance of Y given A ?

- (A) 1
- (B) 1/2
- (C) 1/4
- (D) 2
- (E) None of these

3. (5%) Let the joint probability density function of X and Y be given by

$$f(x, y) = \begin{cases} \sin(x)\sin(y) & \text{if } 0 \leq x \leq \pi/2, 0 \leq y \leq \pi/2; \\ 0 & \text{elsewhere.} \end{cases}$$

Which of the following is the correlation coefficient of X and Y ?

- (A) 0
- (B) 1
- (C) -1
- (D) $\frac{\pi}{4}$
- (E) None of these

4. (5%) Let X have the probability density function

$$f(x) = e^{-x-1}, \quad -1 < x < \infty.$$

Which of the following is correct?

- (A) $\Pr(1 \leq X) = e^{-1}$
- (B) The moment-generating function of X is $M(t) = e^{-t}$
- (C) $E[X] = -1$
- (D) $\text{Var}[X] = 1$
- (E) The distribution function of X is $F(x) = -e^{-(1+x)}, -1 < x < \infty$

5. (5%) Let X and Y have the joint probability density function

$$f(x, y) = 2, \quad 0 \leq x \leq y \leq 1.$$

Which of the following is correct?

- (A) $E[X] = 1$
- (B) $E[Y] = 1/2$
- (C) $E[Y^2] = 1/2$
- (D) $\Pr(0 \leq X \leq Y, 0 \leq Y \leq \frac{1}{2}) = 1/2$
- (E) None of these

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6. (5%) Let the joint probability mass function of X and Y be defined by

$$f(x, y) = \frac{x+y}{32}, \quad x = 1, 2, y = 1, 2, 3, 4.$$

Which of the following is wrong?

- (A) $\Pr(X > Y) = 3/32$
- (B) $\Pr(Y = 2X) = 9/32$
- (C) $\Pr(X + Y = 3) = 3/16$
- (D) $\Pr(X \leq 3 - Y) = 1/4$
- (E) X and Y are independent

7. (5%) Let X_1, X_2, X_3 represent the independent failure times in years of three components in parallel. The probability density functions are $f_{X_1}(x_1) = 3x_1^2, 0 < x_1 < 1$, $f_{X_2}(x_2) = 4x_2^3, 0 < x_2 < 1$, and $f_{X_3}(x_3) = 6x_3^5, 0 < x_3 < 1$. Let $Y = \max(X_1, X_2, X_3)$. Which of the following is correct?

- (A) $\Pr(y = 1/2) = \frac{13}{4096}$
- (B) $\Pr(y = 1/2) = \frac{1}{8192}$
- (C) $\Pr(y \leq 1/3) = 13 \left(\frac{1}{3}\right)^{12}$
- (D) $\Pr(y \leq 1/3) = 1 - 13 \left(\frac{1}{3}\right)^{12}$
- (E) None of these

8. (5%) Let X be a random variable with moment-generating function $M_X(t), -h < t < h$. Which of the following is correct?

- (A) $\Pr(X \geq 1) \leq e^{-t}M_X(t), 0 < t < h$
- (B) $\Pr(X \geq -2) \leq e^{-2t}M_X(t), 0 < t < h$
- (C) $\Pr(X \leq 1) \leq 1 - e^{-t}M_X(t), -h < t < 0$
- (D) $M_X(t) + M_X(6t), -h < t < h$, is also a moment-generating function of some random variable
- (E) None of these

9. (5%) Let X and Y be independent normal random variables with mean 0 and variance 1.

Define a new random variable Z by

$$Z = \begin{cases} X & \text{if } XY > 0; \\ -X & \text{if } XY < 0. \end{cases}$$

Which of the following is correct?

- (A) Z and Y are independent
- (B) $E[Z] = 1$
- (C) $Z = 0$
- (D) Z has a normal distribution
- (E) None of these

10. (5%) Which of the following cannot be cumulative distribution function (CDF)?

- (A) $\frac{1}{2} + \frac{1}{\pi} \tan^{-1}(x), x \in (-\infty, \infty)$
- (B) $(1 - e^{-x})^{-1}, x \in (-\infty, \infty)$
- (C) $e^{-e^{-x}}, x \in (-\infty, \infty)$
- (D) $1 - e^{-x}, x \in (0, \infty)$
- (E) None of these

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二、問答計算題：

1. (25%) Consider two discrete random variables X and Y with joint pmf:

$P(x, y)$	$X = -1$	$X = 0$	$X = 1$
$Y = 2$	0.15	0.15	0.1
$Y = 4$	0.05	0.1	0.15
$Y = 6$	0.1	0.15	0.05

- (5%) Find the marginal distribution of X
- (5%) Find the conditional distribution of X given $Y = 4$
- (5%) Find the conditional mean $E[X|Y]$
- (5%) Are X and Y independent? Prove it or disprove it.
- (5%) Are X and Y uncorrelated? Prove it or disprove it.

2. (25%) Let X be a random variable with PDF

$$f_X(x) = \begin{cases} xe^{-x^2/2}, & \text{if } X > 0; \\ 0 & \text{otherwise.} \end{cases}$$

Suppose that $Y = X^2$. Answer the following questions.

- (5%) Find the PDF of Y
- (5%) Find the mean value $E[Y]$
- (5%) Find the moment generating function of Y
- (10%) Derive the n -th moment $E[Y^n]$