

# 國立中山大學 109 學年度 碩士暨碩士專班招生考試試題

科目名稱：工程數學【資工系碩士班乙組】

## —作答注意事項—

考試時間：100 分鐘

- 考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
- 答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，不得另攜帶紙張，請衡酌作答。
- 答案卡請以 2B 鉛筆劃記，不可使用修正液（帶）塗改，未使用 2B 鉛筆、劃記太輕或污損致光學閱讀機無法辨識答案者，其後果由考生自行負擔。
- 答案卷（卡）應保持清潔完整，不得折疊、破壞或塗改應考證號碼及條碼，亦不得書寫考生姓名、應考證號碼或與答案無關之任何文字或符號。
- 可否使用計算機請依試題資訊內標註為準，如「可以」使用，廠牌、功能不拘，唯不得攜帶具有通訊、記憶或收發等功能或其他有礙試場安寧、考試公平之各類器材、物品（如鬧鈴、行動電話、電子字典等）入場。
- 試題及答案卷（卡）請務必繳回，未繳回者該科成績以零分計算。
- 試題採雙面列印，考生應注意試題頁數確實作答。
- 違規者依本校招生考試試場規則及違規處理辦法處理。

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科目名稱：工程數學【資工系碩士班乙組】

題號：434002

※本科目依簡章規定「不可以」使用計算機(選擇題)

共 2 頁第 1 頁

※All the following questions are True/False questions. Please select "A" for "True" and "B" for "False."

**I. (10%) Discrete Fourier Transform (DFT) is a widely used transformation method in signal processing. According to DFT matrix, please determine the correctness of the following descriptions.**

1. (2%) All circulant matrices are made diagonal by the Discrete Fourier Transform (DFT), regardless of the generating vector  $x$ .
2. (2%) DFT matrix is not a kind of symmetric matrix.
3. (2%) DFT matrix is an unitary matrix.
4. (2%) The DFT is a linear transform.
5. (2%) The orthogonality is not the property of the DFT matrix.

**II. (10%) The four eigenvalue of a  $4 \times 4$  matrix  $A$  are 1, 1, 2, and 3. Please determine the correctness of the following descriptions.**

6. (2%) The determinant of  $A$  is 6.
7. (2%) The trace of  $A$  is 7.
8. (2%) The  $\text{rank}(A-I)=3$ , where  $I$  is the  $4 \times 4$  identity matrix.
9. (2%) The determinant of the adjoint matrix  $A^+$  is 6.
10. (2%) The matrix  $A$  is a simple matrix.

**III. (10%) If matrix  $A$  and matrix  $B$  are row equivalent. Please determine the correctness of the following descriptions.**

11. (2%) The row space of  $A$  is equal to the row space of  $B$ .
12. (2%) The column space of  $A$  is equal to the column space of  $B$ .
13. (2%) The null space of  $A$  is equal to the null space of  $B$ .
14. (2%) The  $\text{rank}(A)$  is equal to the  $\text{rank}(B)$ .
15. (2%) The determinant of  $A$  is equal to the determinant of  $B$ .

**IV. (20%) Please read the following True/False questions and determine the correctness of each description.**

16. (2%) If  $A$  and  $B$  are invertible  $2 \times 2$  matrices, then  $(AB)^{-1} = A^{-1}B^{-1}$ .
17. (2%) If  $A$  is a  $3 \times 3$  matrix such that the system  $Ax = 0$  has only the trivial solution, then the equation  $Ax = b$  is consistent for every  $b$  in  $\mathbb{R}^3$ .
18. (2%) If  $P$  and  $D$  are  $n \times n$  matrices, then  $\det(PDP^{-1}) = \det(D)$ .
19. (2%) If  $T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ 0 \end{bmatrix}$ , then  $\text{Null}(T) = \text{span} \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}$ .
20. (2%) If  $A$  and  $B$  are any  $2 \times 2$  matrices, then  $AB = BA$ .
21. (2%) The matrix  $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & 1 \\ 0 & 3 & 0 \end{bmatrix}$  is not invertible.
22. (2%) The matrix of the linear transformation  $T$  which reflects points about the  $x$ -axis and then about the  $y$ -axis is the same as the matrix of the linear transformation  $S$  which rotates points about the origin by 180 degrees counterclockwise.
23. (2%) If  $V$  is a set that contains the 0-vector, and such that whenever  $u$  and  $v$  are in  $V$ , then  $u + v$  is in  $V$ , then  $V$  is a vector space.
24. (2%) If  $A$  and  $B$  are square matrices, then  $(A + B)^{-1} = A^{-1} + B^{-1}$ .
25. (2%) If  $A$  and  $B$  are square matrices, then  $\det(A + B) = \det(A) + \det(B)$ .

試題請隨卷繳回，請留意背面是否有題

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V. (50%) Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be the 20-periodic function such that

$$f(t) = \begin{cases} -1, & \text{if } -10 < t \leq -3 \\ t^{50}, & \text{if } 3 < t \leq 10 \end{cases}$$

and let

$$f(t) \approx \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos \frac{\pi n t}{10} + b_n \sin \frac{\pi n t}{10} \right)$$

be the general Fourier Series of  $f$ . Please determine the correctness of the following descriptions.

26. (10%) For every  $t \in (-20, 55)$ , the following series can be converged.

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos \frac{\pi n t}{10} + b_n \sin \frac{\pi n t}{10} \right)$$

27. (10%) For every integer  $n \geq 1$ , we have

$$b_n = \frac{1}{10} \int_{-7}^{97} f(t) \sin \frac{\pi n t}{10} dt$$

28. (10%) For every integer  $n \geq 1$ , we have

$$b_n = \frac{2}{10} \int_0^{10} f(t) \sin \frac{\pi n t}{10} dt$$

29. (10%) For every  $t \in (-2, 3)$ , we have

$$0 = \sum_{n=1}^{\infty} \frac{\pi n}{10} \left( -a_n \sin \frac{\pi n t}{10} + b_n \cos \frac{\pi n t}{10} \right)$$

30. (10%) We have

$$-1 = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos \frac{42\pi n}{10} + b_n \cos \frac{42\pi n}{10} \right)$$