

國立中山大學 109 學年度 碩士暨碩士專班招生考試試題

科目名稱：離散數學【資工系碩士班甲組】

— 作答注意事項 —

考試時間：100 分鐘

- 考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
- 答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，不得另攜帶紙張，請衡酌作答。
- 答案卡請以 2B 鉛筆劃記，不可使用修正液（帶）塗改，未使用 2B 鉛筆、劃記太輕或污損致光學閱讀機無法辨識答案者，其後果由考生自行負擔。
- 答案卷（卡）應保持清潔完整，不得折疊、破壞或塗改應考證號碼及條碼，亦不得書寫考生姓名、應考證號碼或與答案無關之任何文字或符號。
- 可否使用計算機請依試題資訊內標註為準，如「可以」使用，廠牌、功能不拘，唯不得攜帶具有通訊、記憶或收發等功能或其他有礙試場安寧、考試公平之各類器材、物品（如鬧鈴、行動電話、電子字典等）入場。
- 試題及答案卷（卡）請務必繳回，未繳回者該科成績以零分計算。
- 試題採雙面列印，考生應注意試題頁數確實作答。
- 違規者依本校招生考試試場規則及違規處理辦法處理。

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科目名稱：離散數學【資工系碩士班甲組】

題號：434004

※本科目依簡章規定「不可以」使用計算機(問答申論題)

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There are 9 problems in this test. Note that you should write down detailed steps for the solution to each problem; otherwise, no credits for that problem will be given.

1. In how many ways can we distributed 10 identical green balls into 5 distinct containers so that
 - (a) [6%] no container is left empty?
 - (b) [6%] the third container has an even number of balls in it?
2. [10%] If $a, b \in \mathbb{Z}^+$, and both are odd, prove that $2|(a^2 + b^2)$ but $4 \nmid (a^2 + b^2)$.
3. Let $|A| = 7$.
 - (a) [6%] How many closed binary operations functions $f: A \times A \rightarrow A$ are there?
 - (b) [6%] How many of these closed binary operations are commutative?
4. [10%] An auditorium has a seating capacity of 900. How many seats must be occupied to guarantee that at least two people seated in the auditorium have the same first and last initials?
5. [10%] In how many ways can 3600 identical envelopes be divided, in package of 25, among five student groups so that each group get at least 150, but not more than 1000, of the envelopes?
6. Find the generating function for the number of partitions of the nonnegative integer n into summands where
 - (a) [6%] each summand must appear an even number of times;
 - (b) [6%] each summand must be even.
7. [10%] If $a_n, n \geq 0$, is the unique solution of the recurrence relation $a_{n+1} - da_n = 0$, and $a_3 = 156/77$, $a_5 = 1628/6336$, what is d ?
8. (a) [6%] How many vertices and how many edges are there in the complete bipartite graphs $K_{m,n}$, where $m, n \in \mathbb{Z}^+$.
(b) [6%] If the graph $K_{m,12}$ has 72 edges, what is m ?
9. [12%] For $a, b, n \in \mathbb{Z}^+$ and $n > 1$, prove that $a \equiv b \pmod{n} \Rightarrow \gcd(a, n) = \gcd(b, n)$.