

# 國立中山大學 109 學年度 碩士暨碩士專班招生考試試題

科目名稱：工程數學【機電系碩士班乙組、丙組】

## — 作答注意事項 —

考試時間：100 分鐘

- 考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
- 答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，不得另攜帶紙張，請衡酌作答。
- 答案卡請以 2B 鉛筆劃記，不可使用修正液（帶）塗改，未使用 2B 鉛筆、劃記太輕或污損致光學閱讀機無法辨識答案者，其後果由考生自行負擔。
- 答案卷（卡）應保持清潔完整，不得折疊、破壞或塗改應考證號碼及條碼，亦不得書寫考生姓名、應考證號碼或與答案無關之任何文字或符號。
- 可否使用計算機請依試題資訊內標註為準，如「可以」使用，廠牌、功能不拘，唯不得攜帶具有通訊、記憶或收發等功能或其他有礙試場安寧、考試公平之各類器材、物品（如鬧鈴、行動電話、電子字典等）入場。
- 試題及答案卷（卡）請務必繳回，未繳回者該科成績以零分計算。
- 試題採雙面列印，考生應注意試題頁數確實作答。
- 違規者依本校招生考試試場規則及違規處理辦法處理。

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科目名稱：工程數學【機電系碩士班乙組、丙組】

題號：438001

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）

共 1 頁第 1 頁

1. Solve the following ODEs.
  - (a)  $2 \cosh x \cos y \, dx = \sinh x \sin y \, dy$  (10%)
  - (b)  $x^2 y'' - 3xy' + 3y = 3 \ln x - 4$  (10%)
2. Solve the following ODE by using the Laplace transform.  
 $y'' + 2y' + 2y = [1 - u(t - 2)]e^t - e^2 \delta(t - 2)$ ,  $y(0) = 0$ ,  $y'(0) = 1$ . (15%)
3. A mixing problem. Each of two tanks contains 200 gal of water, where initially 150 lb (tank  $T_1$ ) and 100 lb (tank  $T_2$ ) of salt are dissolved. The inflow into  $T_1$  is 4 gal/min from  $T_2$ , and 12 gal/min containing 12 lb of salt from the outside. The inflow into  $T_2$  is 16 gal/min from  $T_1$ . The outflow from  $T_2$  is  $4 + 12 = 16$  gal/min. The mixtures are kept uniform by stirring. Find the salt contents  $y_1(t)$  and  $y_2(t)$  in  $T_1$  and  $T_2$ , respectively.
  - (a) Set up the model. (5%)
  - (b) Solve the system. (10%)
4. (a) A line integral with continuous  $F_1, F_2, F_3$  in a domain  $D$  in space. Show that the line integral is path independent if  $\mathbf{F} \cdot d\mathbf{r}$  is exact in  $D$ . (5%)  
 (b) If the differential form (a) is exact in  $D$ , show that  $\text{curl } \mathbf{F} = \mathbf{0}$ . (5%)
5. Is the given function even or odd or neither even nor odd? Find its Fourier series. Show details of your work. (10%)

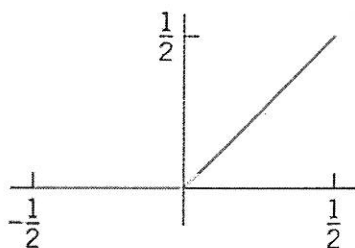


Figure 1.

6. A force  $\mathbf{p} = [4, 2, 0]$  is acting in a line through  $(2, 3, 0)$ . Find its moment vector about the center  $(5, 1, 0)$  of a wheel. (5%)
7. The state of stress in the cube is as follows:
 
$$\sigma_{ij} = \begin{bmatrix} 20 & 40 & 0 \\ 40 & -40 & 0 \\ 0 & 0 & 5 \end{bmatrix} \text{ (in MPa)}$$
 where  $i = 1, 2, 3$ 
  - (a) Determine the principal stresses for the cube. (10%)
  - (b) Determine the directions where the principal stresses occur. (10%)
  - (c) Suppose the compressive/tensile strength of the body are 50 MPa. Does the body fail in the stress state? Explain the reason. (5%)

[Note: Since the stress  $\sigma$  is a symmetric tensor, it has three real eigenvalues  $\sigma_1, \sigma_2, \sigma_3$  called principal stresses, and three corresponding orthonormal eigenvectors called principal directions. The eigenvalue problem can be written as  $\mathbf{t}^{(n)} = \sigma \mathbf{n} = \sigma \mathbf{n}$ , where  $\mathbf{n}$  is a principal direction and  $\sigma$  is a scalar principal stress. Since the traction vector is a multiple of the unit normal,  $\sigma$  is a normal stress component. Thus a principal stress is a stress which acts on a plane of zero shear stress.]