類組：電機類　科目：工程數學A(3003)　　共 2 頁 第 1 頁

所有試題皆為計算題，應詳列計算過程，無計算過程者不予計分。

一、(10%) Please solve $\dfrac{d^2y}{dx^2}-\dfrac{2}{x}\dfrac{dy}{dx}+(\dfrac{2}{x^2}-1)y=4x^2e^x$.

二、(10%) Given $A=\begin{pmatrix}2 & 1\\ 0 & 2\end{pmatrix}$, please find the fundamental matrix e^{At} with Cayley-Hamilton theorem/method of undetermined coefficients.

三、(10%) Suppose $g(s)$ is the Laplace transform of $f(t)$. In other words, $g(s)=L\{f(t)\}$ and $f(t)=L^{-1}\{g(s)\}$. Suppose $g(s)$ is given by $g(s)=\dfrac{\exp(\exp(-s))}{s}$ (note: $\exp(-s)=e^{-s}$), please find $f(4.5)$.

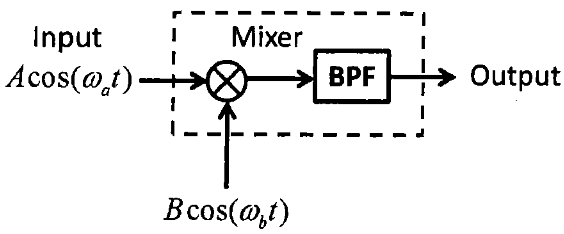
四、(10%) Suppose $f_c(\omega)$ is cosine transform of $f(x)$ and is given by $f_c(\omega)=\dfrac{1}{\sqrt{2a}}\cos\left(\dfrac{\omega^2}{4a}+\dfrac{\pi}{4}\right)$, please find the inverse transform $f(x)$ of $f_c(\omega)$. Hint: you may use the following definite integral:

$$\int_{-\infty}^{\infty}e^{-x^2}dx=\frac{\sqrt{\pi}}{2}$$

五、(12%) A "mixer" is an RF electric circuit that is commonly used in communication systems to shift the EM wave's frequency to a desired frequency band by multiplying an input sinusoidal wave with another sinusoidal wave. The configuration of a mixer is simply a combination of an electric multiplier and a band-pass filter (BPF) as shown in the following diagram. Assume the input vector space is generated by n input sinusoidal waves with the same non-zero amplitude A, but different frequencies, ω_1, ω_2 ..., ω_n, and the output vector space is generated by these n "down-converted" sinusoidal waves. Note: "down-convert" means the frequency is reduced.

(一) (5%) Show that the input n signals are linearly independent.

(二) (7%) Please verify that if this mixer performs a linear transformation in this system.



注意:背面有試題

六、(13%) Fourier series says that any periodic function can be expressed as a linear combination of infinite harmonic cosine and sine functions.

(一) (8%) If the period of a set of periodic functions is 2π, what is the orthonormal basis of the vector space that these periodic functions belong to? You have to show the vectors in the basis are orthonormal to get full credit.

(二) (5%) Find a vector $g(x)$ in a subspace spanned by only $\{1, \cos(x), \cos(2x), \sin(x), \sin(2x)\}$ so that $g(x)$ has the shortest distance to the function $f(x) = x$ in the interval $[-\pi, \pi]$.

Note: In this problem, you may need:

$$\int x \sin ax\,dx = -\frac{x}{a}\cos ax + \frac{1}{a^2}\sin ax \quad \text{and} \quad \int x \cos ax\,dx = \frac{x}{a}\sin ax + \frac{1}{a^2}\cos ax$$

七、(10%) Let a linear equation system $\mathbf{Ax} = \mathbf{b}$ with its coefficient matrix:

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 3 & 2 \end{pmatrix} \qquad \mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

(一) (6%) Please find a lower triangular matrix $\mathbf{L}$ and an upper triangular matrix $\mathbf{U}$ so that matrix $\mathbf{A}$ can be represented as a product of $\mathbf{LU}$.

(二) (4%) Apply the result in (一) to solve the solution of $\mathbf{Ax} = \mathbf{b}$.

八、(10%) Given $f(z) = \frac{z}{|z|}$, find u and v such that $f(z) = u(x, y) + iv(x, y)$; determine where f is differentiable and where f is not.

九、(5%) Expand $\frac{2z}{9+z^2}$ in Laurent series with center z = $3i$ in the neighborhood. Write down at least the first three terms. Determine the precise region of convergence.

十、(10%)

(一) (5%) Find all zeros of $\cos z$ and show that all these are simple zeros.

(二) (5%) Evaluate $\oint_C \frac{z}{\cos z}dz$ where $C: |z| = 5$, counter-clockwise.