

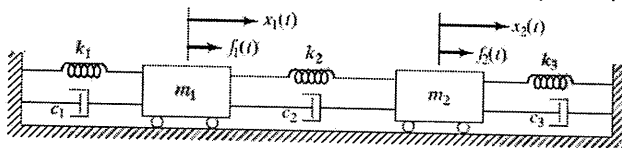
※ 考生請注意：本試題可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Solve the following ordinary differential equations.

(a)  $yy'' = (y')^2$ . (10%)

(b)  $x^2y'' - 2xy' + 2y = x \ln|x|$ ,  $y(1) = 1$ ,  $y'(1) = 0$ . (20%)

2. Consider the two-degree-of-freedom vibration system,



(a) Derive the equation of motion. (10%)

(b) If the forces  $f_1(t) = f_2(t) = 0$ , dampers  $c_1 = c_2 = c_3 = 0$ , masses  $m_1 = 1$ ,  $m_2 = 2$ , spring constants  $k_1 = 1$ ,  $k_2 = k_3 = 2$  and initial conditions  $x_1(0) = 1$ ,  $x_2(0) = 2$ ,  $\dot{x}_1(0) = 2$ ,  $\dot{x}_2(0) = -1$ , find the displacements  $x_1(t)$ ,  $x_2(t)$  of the system. (20%)

3. Evaluate  $\iint_R xy dx dy$ , where  $R$  is the region enclosed by the four parabolae (a)  $y^2 = x$ , (b)  $y^2 = 2x$ , (c)

$x^2 = y$ , and (d)  $x^2 = 2y$ . (20%)

4. (a) Verify that the function  $u(x, y) = 4xy^3 - 4x^3y + x$  is harmonic. (5%)

(b) If the complex function  $f(z) = u(x, y) + iv(x, y)$  is analytic, find the imaginary part  $v(x, y) = ?$  (5%)

(c) Find the derivative  $f'(z)$ . (10%)