

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Solve the non-homogeneous ODE (20%)

$$(2x+1)^2 y'' + 2(2x+1)y' - 16y = 24x - 4$$

2. Using the Laplace transform solves the ODE (20%)

$$y'' + 3y' + 2y = u(t-1) - \delta(t-2); \quad y(0) = 0, \quad y'(0) = 0$$

PS: $u(t-1)$ is a *Unit step function*; $\delta(t-2)$ is a *Delta function*

3. Solve the General solution of P.D.E. (20%)

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}; \quad u(2, t) = u(3, t) = 2$$

4. If $f(t) = (t+1)/2$ and $t \in (0, 1)$, please plot the graph of $f(t)$ and the graphs after Fourier full-range expansion, half-range sine (odd) expansion, and half-range cosine (even) expansion. (20%)

5. For the same-order square matrices A, B, and C, which of the following is incorrect, and please correct the error? (20%)

(a) $(A+B)C = CA + CB$

(b) $(AB)^{-1} = A^{-1}B^{-1}$

(c) $(A+B+C)^T = C^T + B^T + A^T$

(d) $|AB| = |BA| = |A||B|$

(e) $(A+B)^2 = A^2 + 2AB + B^2$

(f) $(kA)^T = k^T A^T$ k is a scalar

(g) $AB \neq BA$

(h) $AB = AC \Rightarrow B = C$