

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Choose the most appropriate answer for the following questions. (40%)

(a) Two sets A and B contains  $a$  and  $b$  elements respectively. If the power set of A contains 16 more elements than that of B, value of ' $b$ ' and ' $a$ ' are respectively

- (1) 4, 5
- (2) 6, 7
- (3) 2, 3
- (4) None of the mentioned

(b) Find the coefficient of  $x^8$  in the expansion of  $(x+2)^{11}$ .

- (1) 640
- (2) 326
- (3) 1320
- (4) 456

(c) For matrix A,  $(A^3) = I$ ,  $A^{-1}$  is equals to:

- (1)  $A^2$
- (2)  $A^{-2}$
- (3) Can't say
- (4) None of the mentioned

(d) If A is an invertible square matrix, then:

- (1)  $(A^T)^{-1} = (A^{-1})^T$
- (2)  $(A^T)^T = (A^{-1})^T$
- (3)  $(A^T)^{-1} = (A^{-1})^{-1}$
- (4) None of the mentioned

(e) If  $f(x) = (x^3 - 1) / (3x + 1)$ , then  $f(x)$  is

- (1)  $O(x^2)$
- (2)  $O(x)$
- (3)  $O(x^2 / 3)$
- (4)  $O(1)$

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(f) What is the recurrence relation for 1, 7, 31, 127, 511?

- (1)  $b_{n+1} = 5b_{n-1} + 3$
- (2)  $b_n = 4b_n + 7!$
- (3)  $b_n = 4b_{n-1} + 3$
- (4)  $b_n = b_{n-1} + 1$

(g) Consider the recurrence relation  $a_1=4$ ,  $a_n = 5n + a_{n-1}$ . What is the value of  $a_{64}$ ?

- (1) 10399
- (2) 23760
- (3) 75100
- (4) 53700

(h) Determine the interval of convergence for  $\sum_{n=0}^{\infty} (x-7)^{n+1} / n^n$ .

- (1)  $-1 < x < 1$
- (2)  $-\infty < x < \infty$
- (3)  $-2 < x < 2$
- (4)  $-1 < x < \infty$

(i) What is the maximum number of edges in a bipartite graph on 14 vertices?

- (1) 78
- (2) 15
- (3) 214
- (4) 49

(j) Let  $D$  be a simple graph on 10 vertices such that there is a vertex of degree 1, a vertex of degree 2, a vertex of degree 3, a vertex of degree 4, a vertex of degree 5, a vertex of degree 6, a vertex of degree 7, a vertex of degree 8 and a vertex of degree 9. What can be the degree of the last vertex?

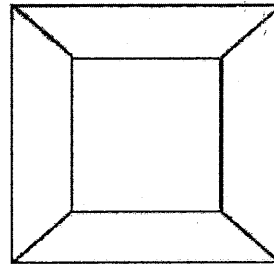
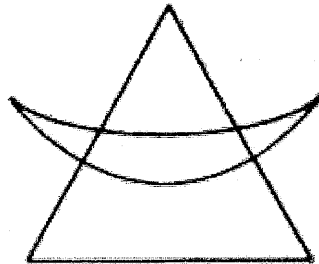
- (1) 4
- (2) 0
- (3) 2
- (4) 5

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2. Consider the  $3 \times 3$  numbered grid below. Each square in the grid will be painted either BLACK or WHITE. The color for each square is decided by tossing a fair coin. Find the probability that the grid does not have a  $2 \times 2$  BLACK square (that is all 4 squares are painted BLACK). (20%)

1	2	3
4	5	6
7	8	9

3. Consider the two figures below which are a child's puzzles. The puzzles expect a child to start from any intersection point and trace each line or curved segment with a colored pencil without raising the pencil or going over any line/curved segment more than once. Can a child solve the puzzles? Justify. (15%)



4. You have a fair die with 6 faces marked 1 to 6. You continue to roll the die repeatedly and only stop when either you roll a 1 or you voluntarily decide to stop at some point. When you stop you get a score that is equal to the value of the last roll. So your last score is either 1 or the value of the last roll before you decided to stop.
- (a) Let  $S(v)$  be the expected score if we stop at value  $v$  or larger. What are the values of  $S(6)$  and  $S(5)$ ? (10%)
- (b) What stopping strategy will you choose to maximize your expected score? (10%)
- (c) If the score was the square of the last rolled value what stopping strategy will maximize your expected score? (5%)