

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. For a differential equation $2xy' + y = 0$
 - a. (2%) Show that $2xy' + y = 0$ can be written as $ydx + 2xdy = 0$
 - b. (4%) Solve the differential equation using separable method
 - c. (2%) Is this differential equation exact?
 - d. (7%) If yes, solve. If not, look for an integrating factor and solve.

2. Solve the initial value problem, $y'' - 6y' + 9y = e^{3x}$,
 - a. (3%) Solve the homogeneous solution
 - b. (5%) Solve the nonhomogeneous solution
 - c. (2%) Solve the initial problem, $y(0) = 1, y'(0) = 1$

3. (15%)
Using Frobenius Method to solve $x^2y'' + (x^2 + x)y' - y = 0$. Try to identify the series as expansions of known functions. Show the details of your work.

4. (20%)
Use Laplace inverse to solve the following equation:

$$Y(s) = \frac{1.151s + 0.1774}{s^4 + 0.739s^3 + 0.921s^2}$$

5. (20%)

$$\frac{\partial^2 T}{\partial x^2} + x = \frac{\partial T}{\partial t} \quad \text{in } 0 < x < 1, \text{ for } t > 0$$

with the boundary conditions of

$$\frac{\partial T}{\partial x} = 0 \quad \text{at } x=0, \text{ for } t > 0$$

$$T = a \quad \text{at } x=1, \text{ for } t > 0$$

and the initial condition of $T = a$ at $t = 0$, in $0 \leq x \leq 1$
 Find the solution of $T(x, t)$.

6. (20%)
Find the finite Fourier cosine transform of the function:
 - a) $f(x) = \begin{cases} 2 & \text{for } 0 \leq x \leq 0.5 \\ -1 & \text{for } 0.5 < x \leq \pi \end{cases} \quad (10\%)$
 - b) $f(x) = e^{-x} \quad 0 \leq x \leq \pi \quad (10\%)$