國立成功大學 109 學年度碩士班招生考試試題

系　所：測量及空間資訊學系

考試科目：線性代數

考試日期：0210，節次：2

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. In the following linear system of equations

$$\begin{aligned} 6x + 2y - \ \ z &= 5 \\ 2x + \ \ y + \ \ z &= 3 \\ -x + \ \ y + 2z &= 1 \end{aligned}$$

the ***coefficient matrix*** A = $\begin{bmatrix} 6 & 2 & -1 \\ 2 & 1 & 1 \\ -1 & 1 & 2 \end{bmatrix}$

(1a) the determinant det A = ? (5%)

(1b) rank A = ? (5%)

(1c) please compute the solution vector by **Gauss elimination.** (10%)

(1d) please compute the inverse A^{-1} by **Gauss-Jordan elimination** (15%)

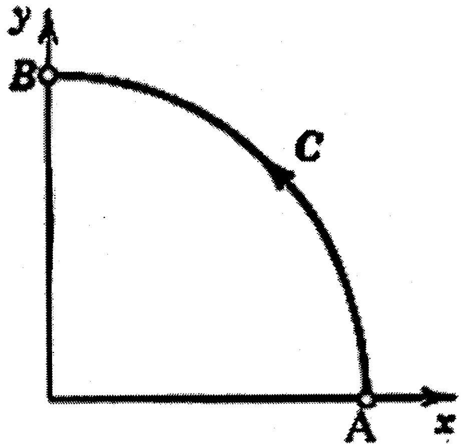
2. Please determine the ①eigenvalues and ②eigenvectors of the matrix A = $\begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$. (15%)

3. Let $\vec{u} = [u_1 \quad u_2 \quad u_3]$ and $\vec{v} = [v_1 \quad v_2 \quad v_3]$ be two vectors. The **inner product** (or **dot product**) $\vec{u} \cdot \vec{v}$ of these two vectors $\vec{u}$ and $\vec{v}$ can also be computed by $\vec{u} \cdot \vec{v}$ $= u_1v_1 + u_2v_2 + u_3v_3$. Please derive this equation $\vec{u} \cdot \vec{v} = u_1v_1 + u_2v_2 + u_3v_3$. (10%)

4.A **line integral** of a vector function $\vec{F}(\vec{r})$ over a curve C is defined by $\int_C \vec{F}(\vec{r}) \cdot d\vec{r} = \int_a^b \vec{F}(\vec{r}(t)) \cdot \frac{d\vec{r}}{dt} dt$. In terms of components, with $d\vec{r}$ = [*dx*, *dy*, *dz*] and ′ = *d/dt*, it becomes $\int_C \vec{F}(\vec{r}) \cdot d\vec{r} = \int_C (F_1 dx + F_2 dy + F_3 dz) = \int_a^b (F_1 x' + F_2 y' + F_3 z') dt$.

Please find the value of the line integral $\int_C \vec{F}(\vec{r}) \cdot d\vec{r} = \int_a^b \vec{F}(\vec{r}(t)) \cdot \frac{d\vec{r}}{dt} dt$ when $\vec{F}(\vec{r})$

$=[-y, -xy]=-y\vec{i} -xy\vec{j}$ and C is the circular arc as in the following figure from A (1,0) to B(0, 1). (20%)



5. After derivation, we have $A = \frac{1}{2}\oint_C (xdy - ydx)$. This formula expresses the area of a closed bounded region R in the xy-plane, whose boundary C consists of finitely many smooth curves, in terms of a line integral over the boundary. Let r and θ be polar coordinates defined by $x = r\cos\theta$, $y = r\sin\theta$.

(5a) Then one can derive A = $\frac{1}{2}\oint_C r^2 d\theta$. Please derive it. (5%)

(5b) Please compute the area of a cardioid $r=a(1-\cos\theta)$, where $0\leq\theta\leq 2\pi$. (10%)

(5c) Please compute the area of a circle $r=a$, where $0\leq\theta\leq 2\pi$. (5%)