

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (30%) Consider a monopoly that serves two distinct markets with their demand functions as $Q_1(p) = a - p$ and $Q_2(p) = k \cdot a - p$ separately. In words, market 2 is k times the size of market 1 where $1 > k > 0$. Suppose the total cost borne by the monopolist is $c(Q) = Q^2/2$ when Q units are produced.
 - a. (10%) If the monopolist has to charge the same price in the two markets, derive the condition under which it is the monopolist's best interest to serve both markets instead of just market 1.
 - b. (10%) Suppose the monopolist is capable of exercising price discrimination. Show that the demand in market 2 is more elastic than that in market 1 at equilibrium.
 - c. (5%) Show that the monopolist is better off by exercising price discrimination over the two markets.
 - d. (5%) Suppose market 2 is accessible to the monopolist only after some additional advertising expenditures $\$b$. Specifically, $k = b^{0.5}$ whenever $\$b$ is spent. Find the optimal advertising spending for the monopolist when it has to charge the same price in the two markets.

2. (30%) Jack has the utility function $u(x, y) = x^a y$ over good x and y where $a > 0$. Suppose the price of good x is $\$p$ while the price of good y is normalized as $\$1$. Jack is endowed with income $\$M$.
 - a. (10%) Derive Jack's Marshallian demand function of the two goods.
 - b. (10%) Derive Jack's Hicksian demand function of the two goods.
 - c. (10%) Verify if the Slutsky equation in terms of elasticity holds for Jack's Marshallian demand function of good y facing the change in the price of good x . That is, $\varepsilon_{p,yx} = \varepsilon_{p,yx}^h + \varepsilon_{w,y} \cdot b_x$, where $\varepsilon_{p,yx}$ is the (uncompensated) price elasticity of good y given the change in price of good x , $\varepsilon_{p,yx}^h$ is the compensated price elasticity, $\varepsilon_{w,y}$ the income elasticity of good y , and b_x the budget share of good x .

3. (25%) Suppose the market demand in a perfectly competitive market is $Q = 500 - 10P$. There are currently 20 identical firms operating in this market and each has the same cost function as $c(q) = 100 + q^2$ when it produces q units of the products.
 - a. (10%) How much is the consumer surplus at the short-run equilibrium of this market?
 - b. (10%) How much deadweight loss will be caused by a specific tax of $\$10$ on the products of this market?
 - c. (5%) Find the number of firms that will survive the long run equilibrium of the market if there's not any specific tax imposed.

4. (15%) Consider an industry of 3 firms. Suppose the market demand is $Q = 200 - P$ and each firm produces with cost function $c(q) = 600 + 40q$ when its output level is q .
 - a. (10%) Find the profit level of each firm at the Nash equilibrium when they engage in quantity competition.
 - b. (5%) Find the deadweight loss caused by the oligopoly when firms compete in quantity.