編號： 194

國立成功大學109學年度碩士班招生考試試題

系　所：電腦與通信工程研究所

考試科目：機率與線性代數

考試日期：0211，節次：3

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (10%) Let X be a random variable with pdf, $f_X(x) = e^{-x}$, $x > 0$. Find pdf of $Y = 1/X^5$.

2. (15%) Let the joint probability density function of X, Y, and Z be given by $f(x, y, z) = 8xyz$, if $0 < x < 1$, $0 < y < 1$, $0 < z < 1$ and $f(x, y, z) = 0$ otherwise. Find correlation coefficients $\rho(X, Y)$, $\rho(X, Z)$, and $\rho(Y, Z)$.

3. (15%) Let X and Y be independent (strictly positive) exponential random variables each with parameter λ. Are the random variables $X + Y$ and X/Y independent?

4. (10%) If X is a random variable with expected value μ and variance σ^2, then for any $t > 0$, prove that $P(|X - \mu| \geq t) \leq \dfrac{\sigma^2}{t^2}$.

5. (20%) Choose the true statement(s) from the following.
   (a) If M is an invertible matrix, then $M + I$ is also an invertible matrix. (I denotes the identity matrix of the size as M).
   (b) For an $n \times n$ matrix A, if $A^2 = O$, where O denotes the zero matrix, then we have $A = O$.
   (c) For an $n \times n$ matrix M, we have $\text{rank}(M^2) \leq \text{rank}(M)$.
   (d) A real-valued square matrix may have complex eigenvalue and complex eigenvectors.

6. Let A be an $n \times n$ real-valued symmetric matrix, $A^T = A$, and I is an identity matrix of size n.
   (a) (20%) Show that $I + A^2$ is always an invertible matrix.
   (b) (10%) Define a transformation from the space of $n \times n$ real-valued matrices to the space of real numbers as $T(A) = \det(A)$, where $\det(A)$ is the determinant of A. Is T a linear transformation?