※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (30%) John Slow is driving from Boston to the New York area, a distance of 180 miles, at a constant speed, whose value is uniformly distributed between 30 miles/hour and 60 miles/hour. Find the probability density function (pdf) of the duration of the trip.

2. (20%) Evaluate $\int_{-\infty}^{\infty}(y-3)^2\,e^{-4(y-2)^2}dy$.

   Hint: Use what you learned in the probability theory.

3. (20%) Choose the true statement(s) from the following.

   (a) If M is an invertible matrix, then $M+I$ is also an invertible matrix. (I denotes the identity matrix of the size as M).

   (b) For an $n \times n$ matrix A, if $A^2 = O$, where O denotes the zero matrix, then we have $A = O$.

   (c) For an $n \times n$ matrix M, we have rank$(M^2) \leq$ rank(M).

   (d) A real-valued square matrix may have complex eigenvalues and complex eigenvectors.

4. Let A be an $n \times n$ real-valued symmetric matrix, $A^T = A$, and I is an identity matrix of size n.

   (a) (20%) Show that $I + A^2$ is always an invertible matrix.

   (b) (10%) Define a transformation from the space of $n \times n$ real-valued matrices to the space of real numbers as $T(A) = \det(A)$, where $\det(A)$ is the determinant of A. Is T a linear transformation?