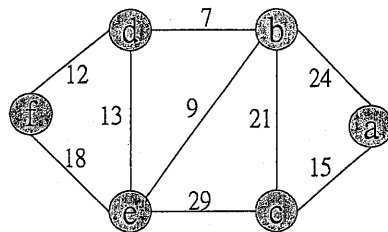


※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

Part I. 資料結構 (50%)

- Answer **True** or **False** for the following statements. Give correct answers for **False** statements. (30%, 6% for each question)

 - In static hashing, the worst-case number of comparisons needed for a successful search is $O(n)$ for open addressing. The number could be reduced to $O(\log n)$ by using balanced search tree.
 - Let d_i be the degree of vertex i in a graph G with $|V| = n$ and $|E| = e$, then $e = \sum_{i=0}^{n-1} d_i$.
 - The path from vertex u to vertex v on a minimal cost spanning tree of an undirected graph G is also a shortest path from u to v .
 - Let G be a graph with e edges and v vertices. If G is represented by adjacency lists, DFS requires $O(ev^2)$ time.
 - If an AOV network represents a feasible project, its topological order is not unique.
- A Bloom Filter consists of m bits of memory and h uniform and independent hash functions f_1, f_2, \dots, f_h . Each f_i hashes a key k to an integer in the range $[1, m]$. Initially all m filter bits are zero, and the data set is empty. When key k is added to the data set, bits $f_1(k), f_2(k), \dots, f_h(k)$ of the filter are set to 1. When a query "Is key k is in the data set?" is made, bits $f_1(k), f_2(k), \dots, f_h(k)$ are examined. The query answer is "maybe" if all these bits are 1. Otherwise, the answer is "no." A *filter error* occurs whenever the answer is "maybe" and the key is not in the data set. Assume that key k is an integer in the range $[1, n]$ and u updates are made. Compute the probability of filter error for an arbitrary query after the u -th update. (10%)
- Consider the following graph:



Compute minimum cost and construct minimum spanning tree. (10%)

Part II. 演算法 (50%)

1. Is $5^{n+1} = O(5^n)$? Is $6^{2n} = O(6^n)$? (10%)
2. Give asymptotic upper and lower bound for $T(n) = \sqrt{n}T(\sqrt{n}) + n$. Assume that $T(n)$ is constant for sufficiently small n . Make your bounds as tight as possible. (10%)
3. Show how the time complexity of quicksort can be made to run in $O(n \lg n)$ time. (10%)
4. Determine an Longest Common Subsequence of $\langle 1,0,0,1,0,1,0,1 \rangle$ and $\langle 0,1,0,1,1,0,1,1,0 \rangle$. (10%)
5. If possible, use the master method to solve $T(n) = 27T\left(\frac{n}{3}\right) + \Theta\left(\frac{n^3}{\lg n}\right)$. (10%)