編號： 251　　國立成功大學 109 學年度碩士班招生考試試題

系　所：企業管理學系

考試科目：微積分　　考試日期：0211，節次：3

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

Part A: Multiple-Choice Questions (40 points, 10 points each)

1. Let

$$f(x) = \frac{-x^2+3}{2x^2-6x}.$$

Then

(a) $y = -\frac{1}{2}$ is a horizontal asymptote;

(b) $x = 0$ is a horizontal asymptote;

(c) $x = 0$ is a vertical asymptote;

(d) None of above.

2. Consider

$$f(x) = 5x^{4/3} - 2x^{5/3}.$$

Then

(a) both $x = 0$ and $x = 8$ are critical numbers of $f(x)$;

(b) $f(x)$ is decreasing on the open interval $(0,8)$;

(c) $f(x)$ are increasing on open intervals $(-\infty, 0)$ and $(8, \infty)$;

(d) $f(0) = 0$ is a relative minimum and $f(8) = 16$ a relative maximum.

3. Consider the improper integral

$$\int_0^\infty e^{-ax} f(x)dx.$$

Let $f(x)$ be continuous on $[0, \infty)$. Assume that there exist $x_0 \geq 0, M > 0$ and $\gamma > 0$ such that $|f(x)| \leq Me^{\gamma x}$ for $x \geq x_0$. Then

(a) the integral is defined (which means that the improper integral converges,) provided $a > 0$;

(b) the improper integral is defined provided $a > \gamma$;

(c) the improper integral is defined provided $a > -\gamma$.

(d) If $f(x) = x$ and the improper integral is defined, the integral equals $-\frac{1}{a^2}$.

4. Which of the following series are convergent?

(a) $\sum_{n=0}^{\infty} \frac{5}{4^n}$; (b) $\sum_{n=0}^{\infty} \frac{n}{\sqrt{n^2+1}}$; (c) $\sum_{n=0}^{\infty} ne^{-n}$; (d) $\sum_{n=0}^{\infty} \frac{(-1)^n}{n+1} x^{n+1}$, for $x \in (0,2)$.

Part B: Please simplify your answers as possible as you can. (60 points)

5. Let $F(x,y) = \ln(x^2 - xy + y^2)$.
   (a) (5 points) Find the domain of F(x,y)
   (b) (5 points) Find the critical point of $F(x,y)$..

6. (10 points) Show that

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{for } x > 0, \\ 0 & \text{for } x \leq 0 \end{cases}$$

gives a density function of a continuous random variable X, for a given $\lambda > 0$.

   (a) (5 points) Verify that the expected value of X, $\mu = E(X) = \int_0^{\infty} x\lambda e^{-\lambda x}\, dx = \frac{1}{\lambda}$,

   (b) (5 points) and the variance of X, $\text{var}(X) = \int_0^{\infty} (x - \mu)^2 \lambda e^{-\lambda x} dx = \frac{1}{\lambda^2}$.

7. (a) (10 points) Estimate $\sqrt[5]{33}$ to 3 decimal places. (Hint: Let $f(x) = \sqrt[5]{x}$, and consider the 1st order Taylor expansion for $f(x + \Delta x)$ at $x = 32$.)
   (b) (5 points) Estimate the possible error for your estimation, and tell us why?

8. (a) (5 points) Show that the sigmoid function $F(x) = \frac{1}{1+e^{-x}}$ defines a continuous probability distribution on $(-\infty, \infty)$;
   (b) (5 points) its density function $f(x) = F'(x)$ is always positive and symmetric about y-axis.
   (c) (5 points) Verify that $f(x)$ reaches its maximum at $x = 0$.

Reference

Ron Larson and Tzuwei Cheng (2014), Calculus: An Application Approach
Bill Armstrong and Don Davis (2014), Brief Calculus for the Business, Social, and Life Sciences