

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Christine has always been weak in mathematics. Based on her performance prior to the final examine in calculus, there is a 40% chance that she will fail the course if she does not have a tutor. With a tutor, her probability of failing decreases to 10%. There is only a 50% chance that she will find a tutor at such short notice. (15%)
 - (1) What is the probability that Christine fails the course? (5%)
 - (2) Christine ends up passing the course. What is the probability that she had found a tutor? (5%)
 - (3) If with a tutor, her probability of failing is still 40%. What is the answer of (2)? (5%)

2. You just arrived a booth of selling delicious fried chicken. There is a long line of waiting costumers. However, you have to leave within 10 minutes so that you can attend a lecture on time. You need probability theory to help you making decision. (15%)
 - (1) If someone tell you that the waiting time follows a uniform distribution between 5 to 20 minutes. What is the probability that you need to wait more than 10 minutes? (5%)
 - (2) If someone tell you that the waiting time follows an exponential distribution with mean 10 minutes. What is the probability that you need to wait more than 10 minutes? (5%)
 - (3) Follow the assumption in (2). If you have to wait 15 minutes, what is the probability that you attend the lecture on time? (5%)

3. There are 1 white ball and 3 red balls in a black box. You randomly draw two balls simultaneously from the black box first, check the color of these two balls and then put them back to the black box. If you obtain no white ball then you win the game and otherwise you lose. (20%)
 - (1) If you play the game once, what is the probability of wining? (5%)
 - (2) If you play this game 3 times, what is the probability that you win 2 games? (5%)
 - (3) For each game, if you win then I pay you 2 dollars and if you lose then you pay me 1 dollar. What is the expected earning for you to play this game? Will you play this game with me? Why? (5%)
 - (4) Following (1) and (3), if you play this game until the first time you win, what is your expected earning? (5%)

4. For the recent presidential election, one observer claimed that the difference of the percentages of votes between two major camps is about 3% to 5%. According to this statement, what is the sample size you would like to choose in order to make a meaningful survey to conclude final percentages of votes? (The 97.5, 98.5 and 99 percentiles of the standard normal is 1.96, 2.17, and 2.33, respectively.) (5%)

5. A production line operation is tested for filling weight accuracy using the following hypotheses $H_0: \mu = 16$ vs $H_a: \mu \neq 16$ where μ denotes the population mean of the filling weight. The sample size is 16 and the population standard deviation is 1. Use Type I error rate $\alpha = 0.05$ to make decision. Recall that the 95 and 97.5 percentiles of the standard normal are 1.645 and 1.96. (10%)

(1) What would a Type II error mean in this situation? (5%)

(2) If the true filling weight is 15.51, what is the Type II error rate for this problem? (5%)

6. A scientist claims that eating a bowl of oat per day can effectively remove bad cholesterol from human blood vessel. In order to ensure this statement, please answer the following questions. (15%)

(1) Please design an experiment (data collection scheme) to examine the statement of this scientist. Describe your experiment in detail. (5%)

(2) What are the null and alternative hypotheses of your experiment? (5%)

(3) What is the corresponding test statistic and the sampling distribution of the test statistic under the null hypothesis? (5%)

7. Consider the following data for two variables, x and y . (20%)

x	9	32	18	15	26
y	10	20	21	16	22

(1) Develop an estimated regression for the data of the form $\hat{y} = b_0 + b_1x$. (5%)

(2) What is the coefficient of determination R^2 for this dataset? (5%)

(3) Predict the value of y when $x=20$. Is this prediction reasonable? Why? (5%)

(4) Predict the value of y when $x=40$. Is this prediction reasonable? Why? (5%)