

※ 考生請注意：本試題可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

Please provide complete and detailed calculations. If only answers are provided, then no credits are to be given.

A. (20%)

Suppose that A and B are independent events associated with an experiment. If the probability that A or B occurs equals 0.6. The probability that A occurs equals 0.4. What is the probability that B occurs.

B. (20%)

Suppose that we have two urns, 1 and 2, each with two drawers. Urn 1 has a gold coin in one drawer and a silver coin in the other drawer. Urn 2 has a gold coin in each drawer. One urn is chosen at random; then a drawer is chosen at random from the chosen urn. The coin found in this drawer turns out to be a gold coin. What is the probability that the coin came from urn 2?

C. (20%)

A, B, and C are three events. Prove $P(A \cap B \cap C) = P(A|B \cap C)P(B|C)P(C)$.

D. (20% with 10% each)

1. Suppose we have a sample of 100 women, of whom 30 have breast cancer and 70 do not. If the probability for a woman to have breast cancer is p , then what is the likelihood of the sample given p ?
2. Suppose we have n independent observations x_1, \dots, x_n from a normal distribution with mean = μ and variance = σ^2 . What is the likelihood of the sample?

E. (20% with 10% each)

Suppose that the joint probability density function of the two-dimensional random variable (X, Y) is given by $f(x, y) = x^2 + \frac{xy}{3}$ for $0 < x < 1, 0 < y < 2$; $f(x, y) = 0$, elsewhere.

Compute the following:

1. $P(X > 0.5)$
2. $P(Y < X)$