※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (25%) Evaluate the following limits:

   (a) $\lim_{x \to 1} \frac{x + x^2 + \cdots + x^n - n}{x - 1}$.

   (b) $\lim_{x \to +\infty} \frac{\sin x}{\sqrt{x^2}}$.

   (c) $\lim_{x \to +\infty} \ln \frac{[x]}{x+1}$, where $[x]$ is the largest integer that is less than or equal to x.

   (d) $\lim_{x \to 0} \frac{x^2 \sin \frac{1}{x}}{\sin x}$.

   (e) $\lim_{t \to 1} \frac{1}{\ln t} \int_t^1 \frac{\sin x}{x} \, dx$.

2. (15%) Find the following integrals:

   (a) $\int_0^2 \frac{1}{1 + e^x} \, dx$.

   (b) $\int_e^{+\infty} \frac{1}{x \ln^2 x} \, dx$.

   (c) $\int_0^1 \int_x^1 \frac{\sin y}{y} \, dy \, dx$.

3. (10%) Compute $\int_0^2 \lim_{n \to \infty} \frac{(2-x)(x+x^n)}{1+x^n} \, dx$.

4. (15%) Which of the following series converge, and which diverge? Give reasons for your answer.

   (a) $\sum_{k=1}^{\infty} (-1)^k \frac{\ln k}{k}$　　　(b) $\sum_{k=1}^{\infty} \frac{k^2}{5(k+1)(k+2)}$　　　(c) $\sum_{k=1}^{\infty} \frac{10 + 5^k}{9 + 6^k}$.

5. (20 %) Sketch the graph of the function $f(x) = \frac{x}{x^2+1}$, $x \in \mathbb{R}$, showing all asymptotes, first and second derivatives, critical points, points of inflection, the intervals on which it increases or decreases, and the intervals on which the graph is concave up or concave down.

6. (15 %) Find the following $\frac{\partial f}{\partial x}$.

   (a) $f(x) = \int_0^{x^2} \sqrt{t^4 + x^3} \, dt$.

   (b) $f(x, y, z) = \ln xy + \ln yz + \ln zx$.

   (c) $z = f(x, y)$ and $xyz = \cos(x + y + z)$.