注意：考試開始鈴響前，不得翻閱試題，並不得書寫、畫記、作答。

國立清華大學 109 學年度碩士班考試入學試題

系所班組別：生命科學院
丙組

科 目 代 碼 ：0604

考 試 科 目 ：計算機概論(演算法與計算機數學)

—作答注意事項—

1. 請核對答案卷（卡）上之准考證號、科目名稱是否正確。
2. 作答中如有發現試題印刷不清，得舉手請監試人員處理，但不得要求解釋題意。
3. 考生限在答案卷上標記「由此開始作答」區內作答，且不可書寫姓名、准考證號或與作答無關之其他文字或符號。
4. 答案卷用盡不得要求加頁。
5. 答案卷可用任何書寫工具作答，惟為方便閱卷辨識，請儘量使用藍色或黑色書寫；答案卡限用 2B 鉛筆畫記；如畫記不清（含未依範例畫記）致光學閱讀機無法辨識答案者，其後果一律由考生自行負責。
6. 其他應考規則、違規處理及扣分方式，請自行詳閱准考證明上「**國立清華大學試場規則及違規處理辦法**」，無法因本試題封面作答注意事項中未列明而稱未知悉。

國立清華大學 109 學年度碩士班考試入學試題

系所班組別：生命科學院丙組

考試科目（代碼）：計算機概論(演算法與計算機數學)(0604)

1. (25%) Let A be a matrix and $\vec{c}$ be a column vector defined as follows.

$$A=\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix},\ \vec{c}=\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}.$$

Answer the following questions.

(a) (5%) Compute $\vec{c}^{t}A\vec{c}$.

(b) (5%) Compute the eigenvalues of A.

(c) (10%) Find the spectrum decomposition of A.

(d) (5%) Find the singular values of A.

2. (25%) An LU-decomposition algorithm and back substitution method is frequently used to solve a linear system of equations. In a computer implementation of LU-decomposition, we can take an n by n matrix $A(i,j), 1 \le i,j \le n$ as an input array and store the results in the same array. Such an algorithm is listed as follows.

```
for i = 1, 2, ⋯ , n − 1
  for k = i + 1, i + 2, ⋯ , n
      r = A(k, i)/A(i, i)  by assuming  A(i, i) ≠ 0,
      A(k,i)=r;
      for j = i + 1, i + 2, ⋯ , n
          A(k, j) = A(k, j) − r ∗ A(i, j);
      endfor
  endfor
endfor
```

Take a matrix $A=\begin{bmatrix} 2 & 1 & 1 \\ 4 & -6 & 0 \\ -2 & 7 & 2 \end{bmatrix}$ as input, what are the contents of A after an implementation of the above algorithm.

國立清華大學 109 學年度碩士班考試入學試題

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3. (10%) Answer the following questions.

(a) (3%) Evaluate $\sum_{k=0}^{4} k!$

(b) (3%) $\sum_{k=0}^{n} \binom{n}{k} 2^k = ?$

(c) (4%) $\sum_{k=0}^{6} \binom{6}{k}\binom{6}{k} = ?$

4. (25%) Let X_1, $X_1, \cdots, X_n$ be a random sample of an exponential distribution with the probability density function $f(x) = \frac{1}{2}e^{-x/2}$, $x \geq 0$.

Define $Y = X_1 + X_2 + \cdots + X_n$.

(a) (5%) Compute the expectation, $E(X_1)$ and the variance, $Var(X_1)$.

(b) (5%) What is the moment-generating function for X_n?

(c) (5%) What is the moment-generating function of Y?

(d) (5%) Compute the expectation, E(Y) and the variance, Var(Y).

(e) (5%) Name the distribution of Y.

5. (15%) Let the random variable Z have the standard normal distribution with the probability density function $f(x) = \frac{1}{\sqrt{2\pi}}e^{-x^2/2}, -\infty < x < \infty$. Define a new random variable Y=2Z+3.

(a) (5%) Compute the expectation, E(Y) and the variance, Var(Y).

(b) (5%) Write down the probability density function of Y.

(c) (5%) Find the moment-generating function of Y.