注意：考試開始鈴響前，不得翻閱試題，並不得書寫、畫記、作答。

國立清華大學 109 學年度碩士班考試入學試題

系所班組別：經濟學系

科目代碼：4503

考試科目：微積分與統計

—作答注意事項—

1. 請核對答案卷（卡）上之准考證號、科目名稱是否正確。
2. 作答中如有發現試題印刷不清，得舉手請監試人員處理，但不得要求解釋題意。
3. 考生限在答案卷上標記「▼由此開始作答」區內作答，且不可書寫姓名、准考證號或與作答無關之其他文字或符號。
4. 答案卷用盡不得要求加頁。
5. 答案卷可用任何書寫工具作答，惟為方便閱卷辨識，請儘量使用藍色或黑色書寫；答案卡限用 2B 鉛筆畫記；如畫記不清（含未依範例畫記）致光學閱讀機無法辨識答案者，其後果一律由考生自行負責。
6. 其他應考規則、違規處理及扣分方式，請自行詳閱准考證明上「**國立清華大學試場規則及違規處理辦法**」，無法因本試題封面作答注意事項中未列明而稱未知悉。

國立清華大學109學年度碩士班考試入學試題

系所班組別：經濟學系 (0545)

考試科目（代碼）：微積分與統計 (4503)

　*請在【答案卷、卡】作答

請依題號作答

第一部分

Please read the following questions carefully, and make sure to show all your work.

1. **[10 points]** Find the limit, if it exists. If the limit does not exist, explain why.
   (a) $\lim_{x\to 1} \frac{x^2-1}{x-1}$
   (b) $\lim_{x\to 2} \frac{x^2+x-6}{|x-2|}$

2. **[10 points]** Differentiate the functions:
   (a) $f(x) = (5x^2 - 2)(x^3 + 3x)$
   (b) $f(x) = \frac{3x+2}{x+1}$

3. **[10 points]** Let $r(x) = f(g(h(x)))$, where $h(1) = 2$, $g(2) = 3$, $h'(1) = 4$, $g'(1) = 3$, $f'(1) = 5$, $g'(2) = 5$, $g'(3) = 7$, and $f'(3) = 6$. Find $r'(1)$.

4. **[10 points]** Evaluate the Integral:
   (a) $\int_{-1}^{5} (1 + 3x)dx$
   (b) $\int_{1}^{4} \frac{2+x^2}{\sqrt{x}} dx$

5. **[10 points]** Use the Lagrange multipliers method to find the extrema of the following problem. Indicate that your answers are the maxima or the minima.

$$f(x, y) = xy \quad \text{subject to} \quad x^2 + y^2 = 2a^2$$

國立清華大學109學年度碩士班考試入學試題

系所班組別：經濟學系 (0545)

考試科目（代碼）：微積分與統計 (4503)

請依題號作答

第二部分

Please answer the following questions with clarity in details.

1. **[10 points]** If X has the probability mass function:

$$f(x)=\begin{cases}\frac{1}{2} & \text{for } x=1\\ \frac{1}{3} & \text{for } x=2\\ \frac{1}{6} & \text{for } x=3\end{cases}$$

   (a) What is variance of the distribution of X?
   (b) What is moment generating function of X?

2. **[15 points]** Let X and Y have the joint probability mass function:

$$f(x,y)=\frac{x+2y}{18},\ x=1,2,\ y=1,2.$$

   (a) Compute $\mathrm{Cov}(X,Y)$.
   (b) Determine ρ, the correlation coefficient.
   (c) Are X and Y independent?

3. **[10 points]** Christian Yelich's baseball batting average in 2019 was 160 hits out of 490 (about 0.33). By De Moivre-Laplace theorem, for the probability of at least 160 hits occurring "by chance" if Yelich's actual batting rate was 0.3:
   (a) What is the distribution for this probability?
   (b) How much is it?

4. **[15 points]** Give the definitions of the following terms:
   (a) Convergence in probability.
   (b) Convergence in distribution.
   (c) Whether convergence in distribution implies convergence in probability, or vice versa, or neither implies the other?

－以下空白－