1.(a) Matrix A shows the measurement of 2 variables for each of 6 samples. Plot the data in the Variable1-Variable2 plane and calculate the "sample covariance matrix", $S = {AA^T}/{(n-1)}$. [10 points]

$$A = \begin{bmatrix} 3 & -4 & 7 & 1 & -4 & -3 \\ 7 & -6 & 8 & -1 & -1 & -7 \end{bmatrix}$$

1.(b) The greatest variance is on the first principal component and the second greatest variance is on the second principal component. What are the first and second principal component of the matrix S? What are the variances on the first and second principal components? [10 points]

1(c) Calculate the maximum and minimum of the Rayleigh quotient, ${(x^TAA^Tx)}/{(x^Tx)}$? Show their corresponding x of the maximum and minimum. [10 points]

2(a) Given the Laplace's equation $\nabla^2 u = u_{xx} + u_{yy} + u_{zz} = 0$, derive the Laplace's equation in spherical coordinates (r, θ, ϕ), using the chain rule. [10 points]

2(b) Assume that u does not depend on θ. Solve the Laplace's equation in spherical coordinates using the method of separating variables. The boundary condition are $u(R, \phi) = f(\phi)$ and $\lim_{r\to\infty} u(r, \phi) = 0$. [10 points]

3. A mass-spring system consists a spring with a spring constant of k = 2 [N/m] and a mass of m = 1 [kg]. The damping constant is c = 40 [N's/m].

(a) What kind of damping system is it? (over, critical, or under) [5 points]

(b) The mass spring system is acted upon by a unit impulse force (Dirac's Delta function) at time $t = 1$. Use the Laplace transform to solve the output oscillation with initial condition $y(0) = 0$, $y'(0) = 0$. [15 points]

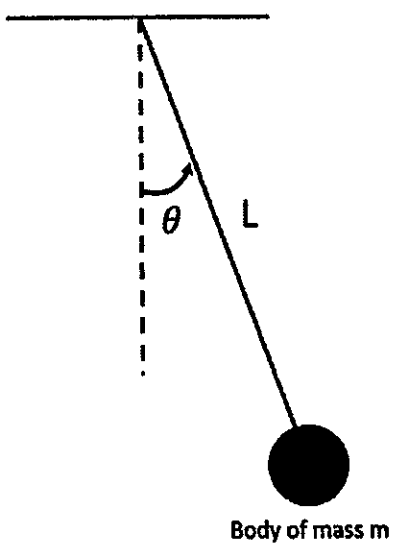
4. Solve the following differential equation: [10 points]

$$(2y - 3xy^2)dx + (3x - 4x^2y^2)dy = 0$$

5. A pendulum consists of a body of mass m and a rod of length L, as shown in the right figure. Neglect air resistance and the weight of the rod. The acceleration of gravity, g, is for the downward gravitational force.

(a) Find the frequency of oscillation of the pendulum when θ is so small that $\sin\theta$ practically equals θ. [10 points]

(b) When θ is not small, the motion of the pendulum becomes a nonlinear system. Show the nonlinear system ODE for the motion of the pendulum and determine the locations and types of the critical points. [10 points]



試題隨卷繳回