1. (20%) Solve for $y(t)$ from the integral equation by Laplace transform

$$y(t) = t + \int_0^t y(\tau)\cos(t-\tau)\,d\tau$$

2. (20%) Find the general solution of the following equation

$$y''' + y' = \sec x$$

3. (20%) Solve the one dimensional wave equation with the initial conditions,

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, \quad -\infty < x < \infty$$

$$\text{I.C.s} \quad \begin{cases} u(x,0) = f(x) \\ u_t(x,0) = g(x) \end{cases}, \quad -\infty < x < \infty$$

4. (20%) Find a matrix $\mathbf{X}$ such that $\mathbf{X}^T\mathbf{A}\mathbf{X} = \mathbf{D}_\lambda$, where $\mathbf{D}_\lambda$ is a diagonal matrix formed by the eigenvalues of $\mathbf{A}$

$$\mathbf{A} = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & 2 \\ 2 & 2 & 3 \end{bmatrix}.$$

5. (20%)What is the Fourier expansion of the periodic function

$$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ \sin x, & 0 < x < \pi \end{cases}$$

and also prove that $\frac{1}{2} + \frac{1}{1x3} - \frac{1}{3x5} + \frac{1}{5x7} - \frac{1}{7x9} + \ldots = \frac{\pi}{4}$

試題隨卷繳回