每題 25 分，4 題共 100 分

1. First, verify that the given function is a solution of the given differential equation, for any constants A, B. Then, solve for A, B so that y satisfies the given initial or boundary conditions.

$$y'' + 4y = 8x^2\,;\ y(x) = 2x^2 - 1 + A\sin 2x + B\cos 2x\,;$$
$$y(0) = 1\,,\ y'(0) = 0$$

2. From the residue theorem of the complex integral calculus, that

$$\int_0^\infty \frac{x^{\alpha-1}}{1+x}dx = \frac{\pi}{\sin\alpha\pi} \quad (0<\alpha<1) \quad \text{show that} \quad \Gamma(\alpha)\Gamma(1-\alpha) = \frac{\pi}{\sin\alpha\pi}. \quad (0<\alpha<1)$$

HINT: $B(p,q) = \int_0^1 x^{p-1}(1-x)^{q-1}dx = \dfrac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$, for $p>0, q>0$;

3. Decompose the given matrix as the sum of two matrices, one symmetric and one skew symmetric.

$$\begin{bmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

4. Let σ be the mass density of a (negligibly thick) distribution of mass over a surface S. That is, σ is the mass per unit area at each point on S; it may vary over S. Then the x, y, z coordinates of the center of gravity are defined as

$$x_C = \frac{1}{M}\iint_S x\sigma\, dA\,,$$
$$y_C = \frac{1}{M}\iint_S y\sigma\, dA\,, \qquad M = \iint_S \sigma\, dA \text{ is the total mass.}$$
$$z_C = \frac{1}{M}\iint_S z\sigma\, dA\,,$$

Evaluate M and x_c in S is the spherical surface $x^2+y^2+z^2=9\,;\sigma = 4+x$.

試題隨卷繳回