1. Factor A into LU, write down the upper triangular system $Ux = c$ which appears after elimination and solve x, for

$$Ax = \begin{bmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 6 & 9 & 8 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 5 \end{bmatrix}. \text{(15\%)}$$

2. Find a basis for each of the four subspaces of $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

(20%)

3. Find a straight line which gives the best fit to the curve $y = x^3+x+1$ between $x = 0$ and $x = 1$ by using orthogonality. (15%)

4. Suppose the 4 by 4 matrix M has four equal rows all containing a, b, c, d. We know that $\det(M) = 0$. The problem is to find $\det(I + M)$ by any method: (10%)

$$\det(I+M) = \begin{vmatrix} 1+a & b & c & d \\ a & 1+b & c & d \\ a & b & 1+c & d \\ a & b & c & 1+d \end{vmatrix}$$

5. Solve $\frac{d\vec{u}}{dt} = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} \vec{u}$, if $\vec{u}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\vec{u}(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, and find e^{At}. (20%)

6. $A = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$ and show A is positive definite. If $\vec{x}^T A\vec{x} = 1$, identify this curve and draw it. (20%)

試題隨卷繳回