考 試 科 目	統計學	系 所 別	風險管理與保險學系精算組	考 試 時 間	2 月 7 日（五）第 4 節

1. Suppose that T_1 and T_2 have the joint density function given by

$$f(t_1, t_2) = e^{-t_1}, \qquad 0 \le t_2 \le t_1 < \infty$$
$$f(t_1, t_2) = 0, \qquad otherwise$$

   (1) Find the probability $P(T_1 < 2, T_2 > 1)$. (10%)
   (2) Find the probability $P(T_1 - T_2 \ge 1)$. (10%)

2. Assume that travelers arrive at a train station at a Poisson rate t. We further assume 2 travelers arrived during the first minute.

   (1) Find the probability that both arrived during the first 20 seconds. (10%)
   (2) Find the probability that at least one arrived during the first 20 seconds. (10%)

3. The following questions are related to the method of maximum likelihood.
   (1) What is the method of maximum likelihood? (10%)
   (2) Let $T_1, T_2, ..., T_n$ be a random sample drawn from a normal distribution with mean μ and variance σ^2. Find the maximum likelihood estimators of mean μ and variance σ^2. (20%)

4. (1) What is the Law of Large Numbers? (10%)
   (2) What is the Central Limit Theorem? (10%)

5. Tom is a candidate for a city counselor election. He sampled 10 voters and would like to test H_0: p= 0.5 against the alternative hypothesis H_a: p< 0.5. The test statistic is T, the number of sampled voters favoring Tom. What is the probability of type I error if he selects a rejection region of $T \le 3$? (10%)

備　　　　註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。