

考試科目	微積分	系所別	政治學系 政治學系(專修班)	考試時間	2月7日(五)第2節
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**Problem 1 (8 points)**

Evaluate the improper integral

$$\int_2^{\infty} \frac{4x^3 + x - 1}{x^2(x-1)(x^2+1)} dx$$

**Problem 2 (8 points)**

Suppose  $f(x)$  has a continuous second derivative  $f''(x)$  for  $x \in (a, b)$ . Find

$$\lim_{h \rightarrow 0} \frac{f(x+2h) - 2f(x) + f(x-2h)}{h^2}$$

**Problem 3 (8 points)**

Find

$$\lim_{x \rightarrow 0} \frac{\int_0^x \left( \int_1^{\cos t} \sqrt{8+u^4} du \right) dt}{x^3}$$

**Problem 4 (8 points)**

Evaluate the integral  $\int_0^{\pi} \sec^2 x dx$ .

**Problem 5 (12 points)**

A number  $a$  is called a fixed point of a function  $f(x)$  if  $f(a) = a$ . Prove that if  $f(x)$  is differentiable and  $f'(x) \neq 1$  for all real number  $x$ , then  $f$  has at most one fixed point.

**Problem 6 (12 points)**

Find the area of the surface obtained by rotating  $y = \sin x$ ,  $0 \leq x \leq \pi$ , about the  $x$ -axis.

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註

- 一、作答於試題上者，不予計分。
- 二、試題請隨卷繳交。

考試科目	經濟學	系所別	經濟學系(組) 國際貿易組	考試時間	2 月 7 日(五) 第 2 節
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**Problem 7 (14 points)**  
Consider the limit

$$\lim_{x \rightarrow \infty} n^2 \sum_{k=1}^n \frac{k}{n^4 + k^4}.$$

- (a) Explain carefully why the limit exists. Express the limit as a definite integral.  
(b) Evaluate this definite integral.

**Problem 8 (18 points)**

- (a) Write down the Maclaurin series of  $\arctan x$ .  
(b) What is the interval of convergence of the above series?  
(c) Find the sum of the series

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)3^n}.$$

**Problem 9 (12 points)**

Determine whether the series is convergent or divergent.

(a)

$$\sum_{n=1}^{\infty} (-1)^n (e^{\frac{1}{n}} - 1).$$

(b)

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^3}.$$

備

註

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二、試題請隨卷繳交。