

考試科目	統計方法	系所別	統計學系	考試時間	2 月 7 日(五) 第四節																				
<p>1. (20%) A box contains 5 black balls and 10 white balls. Each of Player A and Player B is asked to draw 3 balls from the box and record the number of black balls obtained. Each of them does this 100 times, with the following results:</p> <table style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th></th> <th colspan="4">Observed Outcomes</th> </tr> <tr> <th></th> <th>0</th> <th>1</th> <th>2</th> <th>3</th> </tr> </thead> <tbody> <tr> <td>Player A</td> <td>25</td> <td>40</td> <td>25</td> <td>10</td> </tr> <tr> <td>Player B</td> <td>40</td> <td>40</td> <td>15</td> <td>5</td> </tr> </tbody> </table>							Observed Outcomes					0	1	2	3	Player A	25	40	25	10	Player B	40	40	15	5
	Observed Outcomes																								
	0	1	2	3																					
Player A	25	40	25	10																					
Player B	40	40	15	5																					
<p>(a) (10%) Use <math>\alpha = 0.10</math> to test the hypothesis that Player A drew the balls without replacement and Player B drew the balls with replacement.</p> <p>(b) (10%) Use <math>\alpha = 0.10</math> to test the hypothesis that the two multinomial populations are the same.</p>																									
<p>2. (20%) The following data were collected from a training program. 3 persons participated in this study and 3 courses were used to evaluate their resulting performances. The grades are shown below.</p> <table style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th></th> <th>Person 1</th> <th>Person 2</th> <th>Person 3</th> </tr> </thead> <tbody> <tr> <td>Course A</td> <td>84</td> <td>83</td> <td>85</td> </tr> <tr> <td>Course B</td> <td>79</td> <td>77</td> <td>81</td> </tr> <tr> <td>Course C</td> <td>80</td> <td>74</td> <td>77</td> </tr> </tbody> </table>							Person 1	Person 2	Person 3	Course A	84	83	85	Course B	79	77	81	Course C	80	74	77				
	Person 1	Person 2	Person 3																						
Course A	84	83	85																						
Course B	79	77	81																						
Course C	80	74	77																						
<p>(a) (8%) At 95% confidence using the <math>p</math>-value approach, do persons perform differently?</p> <p>(b) (8%) Using the critical value approach and <math>\alpha = 0.05</math>, are there any significant differences among the 3 courses?</p> <p>(c) (4%) A researcher said "one-way ANOVA cannot be applied to the problem in (b)." Do you agree with this comment? Please give your reason.</p>																									
<p>3. (25%) Let <math>(y_1, x_1), (y_2, x_2), \dots, (y_n, x_n)</math>, and <math>(w_1, x_1), (w_2, x_2), \dots, (w_n, x_n)</math>, be two sets of independent observations where <math>x_i, i = 1, \dots, n</math>, are fixed constants. Suppose we fit the model <math>y_i = \alpha_1 + \beta x_i + \epsilon_i</math> to the first data set and the model <math>w_i = \alpha_2 + \beta x_i + \eta_i</math> to the second data set. Suppose that <math>\epsilon_i</math> and <math>\eta_i</math> are independently distributed with mean zero and variance <math>\sigma^2</math>.</p> <p>(a) (15%) Find the least squares estimates of <math>\alpha_1, \alpha_2</math> and <math>\beta</math>.</p> <p>(b) (10%) Following Part (a), how to use t test to test <math>H_0: \beta = 0</math> vs. <math>H_1: \beta \neq 0</math>? Please state your testing procedure(s) clearly to get full credits.</p>																									
備註	<p>一、作答於試題上者，不予計分。 二、試題請隨卷繳交。</p>																								

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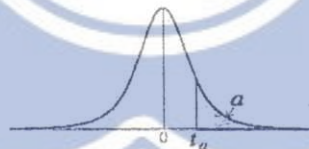
4. (25%) Bowl A contains 100 red balls and 200 white balls; Bowl B contains 200 red balls and 100 white balls. Let  $p$  denote the probability of drawing a red ball from a bowl, but say  $p$  is unknown, since it is unknown whether Bowl A or Bowl B is being used. We shall test the null hypothesis  $H_0: p = 1/3$  against  $H_1: p = 2/3$ .
- (a) (4%) First, we draw three balls at random, one at a time and with replacement, from the selected bowl. Let  $X$  be the number of red balls drawn. Then we decide to reject  $H_0$  if  $X \geq 2$ . Please give a reason to explain the rejection area should be  $\{X \geq 2\}$  rather than  $\{X \leq 2\}$ .
- (b) (6%) Please find the probabilities of making type I and type II errors in (a).
- (c) (15%) As you can see in (b), the errors are too large, so we decide to draw more balls, say  $n$  balls, and the rejection area is  $\{X \geq nr\}$ , where  $X$  is the number of red balls from the  $n$  draws and  $r$  is the proportion and between 0 and 1. Please find the values of  $n$  and  $r$ , so that the type I error probability is 5% and the power is 90%. (Ps. Use the central limit theorem to find the approximate type I and II errors.)

5. (10%) Let  $X_1, X_2, \dots, X_9$  be a random sample from the normal distribution  $N(6, 25)$ , and let  $\bar{X}$  and  $S^2$  denote the sample mean and sample variance respectively. Find each of the following:

- (a) (5%)  $P(1.860 < \frac{3(\bar{X}-6)}{S})$
- (b) (5%)  $P(S^2 \leq 34.4688)$

Critical Values of the  $t$  Distribution

A. table entry is the value of  $t_\alpha$ , having an area to the right of  $\alpha$  under a  $t$  distribution with  $df$  degrees of freedom.



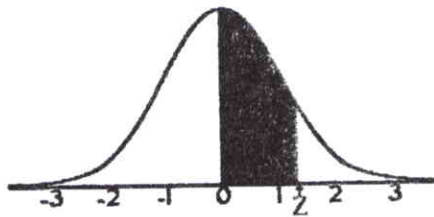
df	$t_{0.20}$	$t_{0.15}$	$t_{0.10}$	$t_{0.05}$	$t_{0.025}$	$t_{0.01}$	$t_{0.005}$	$t_{0.001}$	$t_{0.0005}$
1	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.3	636.6
2	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.33	31.60
3	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.21	12.92
4	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781

備

註

- 一、作答於試題上者，不予計分。  
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### STANDARD NORMAL TABLE (Z)

Entries in the table give the area under the curve between the mean and  $z$  standard deviations above the mean. For example, for  $z = 1.25$  the area under the curve between the mean (0) and  $z$  is 0.3944.

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0190	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2969	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3513	0.3554	0.3577	0.3529	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998

備

註

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T-12 Tables

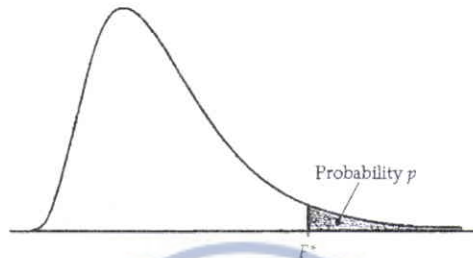


Table entry for  $p$  is the critical value  $F^*$  with probability  $p$  lying to its right.

**TABLE E**

F critical values

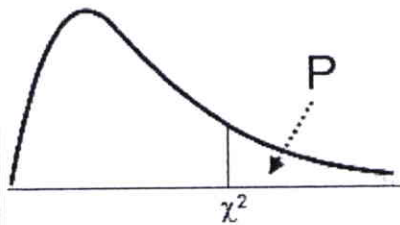
		Degrees of freedom in the numerator								
		1	2	3	4	5	6	7	8	9
Degrees of freedom in the denominator	$p$									
	1	.100	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44
.050		161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54
.025		647.79	799.50	864.16	899.58	921.85	937.11	948.22	956.66	963.28
.010		4052.2	4999.5	5403.4	5624.6	5763.6	5859.0	5928.4	5981.1	6022.5
.001		405284	500000	540379	562500	576405	585937	592873	598144	602284
2	.100	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38
	.050	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38
	.025	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39
	.010	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39
	.001	998.50	999.00	999.17	999.25	999.30	999.33	999.36	999.37	999.39
3	.100	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24
	.050	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
	.025	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47
	.010	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35
	.001	167.03	148.50	141.11	137.10	134.58	132.85	131.58	130.62	129.86
4	.100	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94
	.050	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
	.025	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90
	.010	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66
	.001	74.14	61.25	56.18	53.44	51.71	50.53	49.66	49.00	48.47
5	.100	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32
	.050	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
	.025	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68
	.010	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16
	.001	47.18	37.12	33.20	31.09	29.75	28.83	28.16	27.65	27.24
6	.100	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96
	.050	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
	.025	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52
	.010	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98
	.001	35.51	27.00	23.70	21.92	20.80	20.03	19.46	19.03	18.69
7	.100	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72
	.050	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
	.025	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82
	.010	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72
	.001	29.25	21.69	18.77	17.20	16.21	15.52	15.02	14.63	14.33

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Upper tail	0.3	0.2	0.1	0.05	0.02	0.01	0.005	0.001
df 2	2.41	3.22	4.61	5.99	7.82	9.21	10.60	13.82
3	3.66	4.64	6.25	7.81	9.84	11.34	12.84	16.27
4	4.88	5.99	7.78	9.49	11.67	13.28	14.86	18.47
5	6.06	7.29	9.24	11.07	13.39	15.09	16.75	20.52
6	7.23	8.56	10.64	12.59	15.03	16.81	18.55	22.46
7	8.38	9.80	12.02	14.07	16.62	18.48	20.28	24.32
8	9.52	11.03	13.36	15.51	18.17	20.09	21.95	26.12
9	10.66	12.24	14.68	16.92	19.68	21.67	23.59	27.88
10	11.78	13.44	15.99	18.31	21.16	23.21	25.19	29.59
11	12.90	14.63	17.28	19.68	22.62	24.72	26.76	31.26
12	14.01	15.81	18.55	21.03	24.05	26.22	28.30	32.91
13	15.12	16.98	19.81	22.36	25.47	27.69	29.82	34.53
14	16.22	18.15	21.06	23.68	26.87	29.14	31.32	36.12
15	17.32	19.31	22.31	25.00	28.26	30.58	32.80	37.70
16	18.42	20.47	23.54	26.30	29.63	32.00	34.27	39.25
17	19.51	21.61	24.77	27.59	31.00	33.41	35.72	40.79
18	20.60	22.76	25.99	28.87	32.35	34.81	37.16	42.31
19	21.69	23.90	27.20	30.14	33.69	36.19	38.58	43.82
20	22.77	25.04	28.41	31.41	35.02	37.57	40.00	45.31
25	28.17	30.68	34.38	37.65	41.57	44.31	46.93	52.62
30	33.53	36.25	40.26	43.77	47.96	50.89	53.67	59.70
40	44.16	47.27	51.81	55.76	60.44	63.69	66.77	73.40
50	54.72	58.16	63.17	67.50	72.61	76.15	79.49	86.66

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