國立政治大學 109 學年度 碩士暨碩士在職專班 招生考試試題

考試科目	數理統計學	系所別	統計學系	考試時間	2 月 7 日(五) 第二 節

1. (a) Let $X_1, X_2, \cdots, X_n$ be i.i.d. uniform random variables in the interval $(0,1)$. Find the density function of the range of $(X_1, X_2, \cdots, X_n)$. (10%)

   (b) Let $X_1, X_2, \cdots, X_n$ be i.i.d. random variables with an absolutely continuous monotone increasing distribution function F and also let $(Y_1, Y_2, \cdots, Y_n)$ denote the corresponding ordered statistic. Prove that $F(X_1)$ is distributed uniformly in the interval $(0,1)$ and hence show how you can use the result you have obtained in (a) to get the density function of the random variable $F(Y_n) - F(Y_1)$. (15%)

2. Let $X_1, X_2, \cdots, X_n$ be i.i.d. random variables with pdf

$$f(x) = \frac{1}{6}\theta^4 e^{-\theta x} x^3; x \geq 0, \theta > 0.$$

   (a) Show that $\frac{3}{X_1}$ is an unbiased estimator of θ. (10%)

   (b) Find the joint distribution of X_1 and $\sum_{i=1}^n X_i$ and hence find the conditional density of X_1 given $\sum_{i=1}^n X_i$. (10%)

   (c) Show that the UMVUE of θ is $E\left(\frac{3}{X_1} \middle| \sum_{i=1}^n X_i\right)$ and compute the conditional expectation.. (10%)

   (d) Find the Cramer-Rao lower bound for the variance of an unbiased estimator of θ. Does the variance of UMVUE attain the Cramer-Rao lower bound? (10%)

   (e) Show that $\frac{X_1}{\sum X_i}$ and $\sum X_i$ are independent random variables and hence show that $E\left(\frac{X_1}{\sum X_i}\right) = \frac{1}{n}$. (10%)

3. Suppose that $X_1, X_2, \cdots, X_n$ be i.i.d. Poisson random variables with parameter λ_1. Independent variables $Y_1, Y_2, \cdots, Y_n$ are i.i.d. Poisson with parameter λ_2.

   (a) Show that the conditional distribution of $\sum X_i$, given that $\sum X_i + \sum Y_i = l$, is Binomial$\left(l, \frac{\lambda_1}{\lambda_1 + \lambda_2}\right)$. (10%)

   (b) How could you use the binomial distribution in (a) to test $H_0: \lambda_1 = \lambda_2$ vs. $H: \lambda_1 \neq \lambda_2$? (15%)

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。