考試科目	計算機數學	系所別	資訊科學系	考試時間	2月7日(五) 第4節

I. 離散數學: 六大題(1-6)，共 60%

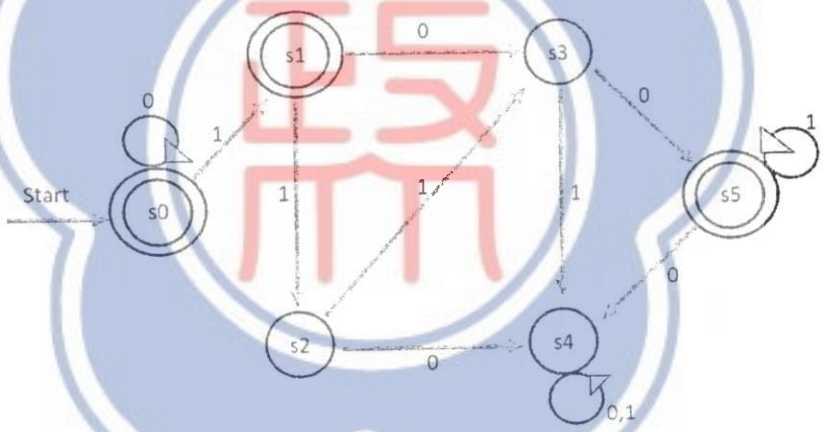
II. 線性代數: 三大題(7-9)，共 40%

請書寫必要的解題過程，僅書寫答案而缺乏必要的過程，無法獲得該題滿分。可使用中文或英文作答，力求書寫工整，如字跡潦草，無法閱讀，將影響評分。

1. (10%) Find the language recognized by the given deterministic finite-state-machine with no output (or called finite-state-automaton). Final states are indicated in state diagrams by using double circles (ie., s0, s1 and s5 are final states). Write down your answer using the presentation like the following example:

$$\{1\}^*\{0\}\{0\}^* \cup \{0\}\{10,11\}\{0,1\}^*.$$

Here the example says that the languages recognized by a finite-state-automaton are either $\{1\}^*\{0\}\{0\}^*$ or $\{0\}\{10,11\}\{0,1\}^*$ where $\{1\}^*\{0\}\{0\}^*$ is the language starting with any number of 1s followed by a 0 and then followed by any number of 0s. And, $\{0\}\{10,11\}\{0,1\}^*$ means the language starts with a 0 and then followed by 10 or 11 and finally ends with $\{0,1\}^*$which means a bit string with arbitrary length.



2. (10%) Use Chebyshev's inequality to find an upper bound on the probability that the number of tails that come up when a biased coin with probability of heads equal to *0.6* is tossed n times deviates from the mean by more than $\sqrt{n}$.
3. (10%) Suppose that we have prior information concerning whether a random incoming message is spam. In particular, suppose that over a time period, we find that s spam messages arrive and h messages arrive that are not spam.
   (a) Use this information to estimate p(S), the probability that an incoming message is spam, and $p(\overline{S})$, the probability an incoming message is not spam.
   (b) Let W be the event that an incoming message contains the word w. Use Bayes' theorem and part (a) to estimate the probability that an incoming message containing the word w is spam, where p(w)=p($W|S$) is the probability that w occurs in a spam message and q(w)= $p(W|\overline{S})$ is the probability that w occurs in

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a message that is not spam.

4. (10%) The concept of equivalence relation is characterized by three properties.
   (a). (5%) What are the three properties?
   (b). (5%) Show that isomorphism of simple graphs is an equivalence relation.
5. (10%) The recurrence relation $a_n = 9a_{n-1} - 26a_{n-2} + 24a_{n-3}$ is a linear homogeneous recurrence relation.
   (a). Find the characteristic root(s) of the recurrence relation.
   (b). Assume the initial conditions of the relation are $a_0 = 4, a_1 = 15, a_2 = 61$. Find the solution to the recurrence relation.
6. (10%)
   (a). (5%) Find the least positive integer x such that $x \equiv 28^{200} mod\ 19$.
   (b). (5%) Use the Extended Euclidean Algorithm to find the least positive integer x such that $13x \equiv 1 \pmod{2436}$.
7. (10%) Maximize z where $z = 25x + 60y$ subject to the constraints
   $$2y - x \le 5, \quad y + 4x \le 25, \quad y + x \ge 7, \quad x \ge 0, \quad y \ge 0.$$
8. (10%) Find a steady state vector for the matrix M where
   $$M = \begin{bmatrix} 0 & 0.5 & 0 \\ 0.5 & 0 & 1 \\ 0.5 & 0.5 & 0 \end{bmatrix}.$$
9. (20%) Let $x_1^2 + x_2^2 + 2x_3^2 + 2x_1x_2 = 2$.
   (a). (5%) Express the equation as a matrix representation of the form $x^tAx=2$. Here $x^t = [x_1 \quad x_2 \quad x_3]$ is the transpose of the matrix x.
   (b). (5%) Find the eigenvalues of this matrix A.
   (c). (5%) Find the orthogonal matrix C that diagonalizes A
   (d). (5%) Reduce the quadratic form $x_1^2 + x_2^2 + 2x_3^2 + 2x_1x_2$ by the Principal-Axes Theorem for R^3.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。