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國立政治大學圖書館

1. Find the derivative of

a. (10 points)  $y = \frac{(1-2x)(3x+2)}{5x-4}$

b. (10 points)  $y = \ln \frac{x(x^2+1)^2}{\sqrt{2x^3+1}}$

2. (10 points) Find the tangent line to the graph of

$$y = \sqrt[3]{(x^2+4)^2}$$

when  $x=2$ .

3. (10 points) Find the slope of the graph of  $2x^2 - y^2 = 1$  at the point (1,1).

4. (10 points) A company is increasing the production of a product at the rate of 200 units per week. The weekly demand function is modeled by

$$p = 100 - 0.001x$$

where  $p$  is the price per unit and  $x$  is the number of units produced in a week. Find the rate of change of the revenue with respect to time when the weekly production is 2000 units.

5. (10 points) By increasing its advertising cost  $x$  for a product, a company discovers that it can increase the sales  $y$  (in thousands of dollars) according to the model

$$y = \frac{1}{10,000}(300x^2 - x^3), \quad 0 \leq x \leq 200$$

Find the point of diminishing returns for this product.

(Hint: An increased investment beyond this point is usually considered a poor

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6. (10 points) The demand and supply functions for a product are modeled by

$$\text{Demand: } p = -0.36x + 9$$

$$\text{Supply: } p = 0.14x + 2$$

Where  $x$  is the number of units (in millions). Find the producer surplus for this product.

7. (10 points) A psychologist finds that the probability that a participant in a memory experiment will recall between  $a$  and  $b$  percent (in decimal form) of the material is

$$P(a \leq x \leq b) = \int_a^b \frac{28}{9} x \sqrt[3]{1-x} dx, \quad 0 \leq a \leq b \leq 1.$$

Find the probability that a randomly chosen participant will recall between 0% and 87.5% of the material.

8. (10 points) Find the indefinite integral  $\int x^2 e^x dx$

9. (10 points) Evaluate the definite integral  $\int_1^e \ln x dx$