

國立政治大學圖書館

1. (12 points) An research wants to estimate the proportion which possess a given characteristic. A random sample of size 225 is taken resulting in 90 items which possess the characteristic. What is 95% confidence interval to estimate the population proportion? Explain the result.
2. (14 points) A student randomly guesses the answers to a four question true/false test. If there is a 50% chance of guessing correctly on each question.
 - (a) What is the probability that the student misses one question?
 - (b) What is the probability that the student misses no question?
3. (14 points) An analysis of personal loans at Mucha Bank revealed the following facts: 15% of all personal loans are in default (D), 85% of all personal loans are not in default (ND), 20% of those in default are homeowners (H | D), and 70% of those not in default are homeowners (H | ND). If one of the personal loans is selected at random,
 - (a) what is $P(H \cap D)$? Explain the result.
 - (b) what is $P(D | H)$? Explain the result.
4. (20 points) Let X_1, X_2, \dots, X_n be n independent random variables having identical distributions with mean μ and variance σ^2 .
 - (a) Please describe the "Central Limit Theorem" using these variables.
 - (b) Please set up an example to demonstrate the application of this theorem.
5. (20 points) Given the following joint probability distribution,

		X		
		0	1	2
Y	0	0.05	0.1	0.03
	1	0.21	0.11	0.19
	2	0.08	0.15	0.08

- (a) Find the marginal distribution of X and Y.
 - (b) Calculate $E[X]$.
 - (c) Calculate $\text{Var}[X]$.
 - (d) Calculate $\text{Cov}[X, Y]$.
 - (e) Calculate the conditional probability $\text{Prob}[X = 1 | Y > 0]$.
6. (20 points) The following data is to be used to construct a linear regression model:

$$\hat{Y}_i = a + bX_i$$

x	3	5	4	7	12
y	10	12	14	18	20

- (a) What is the value of the slope b ?
- (b) What is the value of the intercept a ?
- (c) What is the value of the coefficient of determination R^2 ?
- (d) What is the value of the standard error of the estimate S_e ?
- (e) Using the significance level 0.05 to test the null hypothesis $H_0 : \beta = 0$.

$$(t_{3, 0.025} = 3.182)$$