考試科目 Course	微積分	系級	應數系 碩士班	日期 Date, Period	4月22日 第 節	試題編號 Course No.

第一大題為選擇題 計14小題，對每一小題 請選一個最適當(或最接近)的選項，不需寫計算過程或証明. 每小題3分 答錯不倒扣；第二至第六大題 必須詳細敘述每一步的計算過程或使用的定理。

(42%) 1. The distance from the point $(-1, 4)$ to the circle $x^2+y^2-4x=0$ is

(a) 2 (b) 3 (c) 4 (d) 5 (e) 6

2. Let y be a differentiable function of x such that $2xy+x^2y^2=3$. Which of the following functions, where defined, is equal to $\frac{dy}{dx}$?

(a) $-xy$ (b) $-\frac{xy^2}{1+x^2y}$ (c) $-\frac{x^2y}{1+xy^2}$ (d) $-\frac{x}{y}$ (e) $-\frac{y}{x}$

3. Suppose $\lim_{x\to 1} f(x)=3$. Then for any positive number ε there is a positive number δ such that, whenever $0<|x-1|<\delta$,

(a) $|x-3|<\varepsilon$ (b) $|f(x)-1|<\varepsilon$ (c) $|f(1)-3|<\varepsilon$ (d) $|f(x)-3|<\varepsilon$ (e) $|f(x)-\varepsilon|<3$

4. If $g\left(\frac{1+3x}{2}\right)=x-1$, $-\infty<x<\infty$, then $g\left(\frac{2+5y}{3}\right)$ must equal:

(a) $y-1$ (b) $\frac{5y-1}{3}$ (c) $\frac{3+5y}{2}$ (d) $\frac{1+10y}{9}$ (e) $\frac{10y-8}{9}$

5. If, for all x, $f'(x)=(x-5)^4(x-3)^3$, it follows that the function f has

(a) a relative minimum at $x=3$ (b) a relative maximum at $x=3$

(c) both a relative minimum at $x=3$ and a relative maximum at $x=5$

(d) neither a relative maximum nor a relative minimum

(e) relative minima at $x=3$ and at $x=5$.

6. If $x_n=\begin{cases}\frac{n+3}{3n} & \text{if } n \text{ is odd}\\ \frac{2n+3}{6n} & \text{if } n \text{ is even}\end{cases}$, then $\lim_{n\to\infty} x_n=$

(a) $1/3$ (b) 1 (c) 2 (d) 3 (e) does not exist

7. Let $z=f(u,v)$, where f has continuous first partial derivatives. If $u=2y-x$ and $v=y-x$, then $\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=$

(a) $-\frac{\partial z}{\partial v}$ (b) $\frac{\partial z}{\partial v}$ (c) $\frac{\partial z}{\partial u}-\frac{\partial z}{\partial v}$ (d) $\frac{\partial z}{\partial u}$ (e) 0

8. Which of the following functions does not satisfy the hypotheses of Rolle's theorem on the interval $[0,1]$?

(a) $g(x)=\frac{x^2-x}{2x-1}$ (b) $g(x)=x^2-x$ (c) $g(x)=x^3-x^2$ (d) $g(x)=(x-1)(e^x-1)$ (e) $g(x)=\frac{x^2-x}{x+1}$

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一、9. What is the number of points of discontinuity of the function

$$g(x) = \begin{cases} x + 1/2 & \text{if } x \le 0 \\ \frac{1}{2} & \text{if } 0 < x \le 2 \\ 3 - x & \text{if } 2 < x \le 3 \\ (3-x)^2 & \text{if } 3 < x \end{cases}$$

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

10. If f and g are differentiable functions such that $f(x) = e^{2x} \cdot g(x)$, $g(0) = 2$, and $g'(0) = -1$, then $f'(0) =$

(a) -1 (b) 0 (c) 1 (d) 2 (e) 3

11. If $f(x) = \frac{d}{dx} g(x)$ is integrable on the closed interval $[0,1]$, then $\int_0^1 g(x) f(x)\,dx =$

(a) $\frac{f^2(1) - f^2(0)}{2}$ (b) $\frac{g(1)f(1) - g(0)f(0)}{2}$ (c) $g(1)f(1) - g(0)f(0)$ (d) $g(1) - g(0)$ (e) $\frac{g^2(1) - g^2(0)}{2}$

12. If $\lim_{x \to 0} \frac{f(\sin x + 3x) - f(0)}{x} = \frac{1}{5}$, then $f'(0) =$

(a) $\frac{1}{20}$ (b) $\frac{1}{5}$ (c) $\frac{1}{4}$ (d) $\frac{4}{5}$ (e) 5

13. Let $f(x) = x^3 + 3x$, $0 \le x < \infty$, and let g be the inverse function of f. Then $g'(4) =$

(a) $\frac{1}{72}$ (b) $\frac{1}{30}$ (c) $\frac{1}{6}$ (d) $\frac{1}{4}$ (e) 1

14. If f is differentiable on $(-\frac{1}{2}, \frac{1}{2})$, then $\lim_{h \to 0} \frac{1}{h} \int_0^{h/2} f(x)\,dx$ must equal:

(a) $f'(0)/2$ (b) $f'(0)$ (c) $f(0)/2$ (d) $f(0)$ (e) $\frac{f(0) + f'(0)}{2}$

(20%) 二、Each of the following integrals represents the area of a region in a Cartesian coordinate plane. Sketch the region. Express the area of the region as a double integral with the order of integration reversed. Then evaluate these new integrals.

(1) $\int_0^4 \int_{-\sqrt{4-y}}^{(4-y)/2} dx\,dy$ (2) $\int_{-1}^1 \int_{u^2}^1 dv\,du$

(3) $\int_0^{3/2} \int_{-\sqrt{9-4t^2}}^{\sqrt{9-4t^2}} t\,ds\,dt$ (4) $\int_0^2 \int_0^{4-w^2} 2w\,dz\,dw$

(10%) 三、Use Taylor's formula to find a quadratic polynomial that approximates

$$f(x,y) = \sin x \sin y$$

near the origin. How accurate is the approximation if $|x| \le 0.1$ and $|y| \le 0.1$?

(8%) 四、Prove or disprove that, if $t > 0$, then $t > \ln(1+t)$.

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(18%) 五. A person 5 ft tall walks at the rate of 3 ft/sec away from a streetlight that is 20 ft above the ground. (a) At what rate is the tip of this person's shadow moving? (b) At what rate is the length of this person's shadow changing?

(12%) 六. Test the following series for convergence or divergence. In case of convergence, determine whether the series converges absolutely.

(a) $\sum_{n=1}^{\infty} \frac{(-100)^n}{n!}$, (b) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(1+n)}{n^2}$, (c) $\sum_{n=1}^{\infty} \frac{(-2)^{n+1}}{n+5^n}$.