

考試科目	統計學	所別	財政系	考試時間	月	日	上	午	第	節
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國立政治大學圖書館

1. (21 points) The probability density of the continuous random variable  $X$  is given by

$$f(x) = \begin{cases} \frac{1}{5} - \frac{1}{10}|x - c| & \text{for } 2 < x < 12 \\ 0 & \text{elsewhere} \end{cases}$$

Find

- (a) the values of  $c$ ;
  - (b) the expected value of  $X$ ,  $E(X)$ ;
  - (c) the variance of  $X$ ,  $\text{Var}(X)$ .
2. (21 points) A Gallup poll found that 22% of 200 men and 16% of 300 women surveyed favored a tax reform proposal.
- (a) What is the point estimate of the difference between the two population proportions?
  - (b) Construct a 95% confidence interval for the difference between the two population proportions.
  - (c) With a 5% significance level, can you claim that the proportion of men who favored such a tax proposal was equal to that of women?
3. (21 points) Let  $R^2 = \text{SSR}/\text{SST}$  denote the coefficient of determination for the sample regression line of  $Y$  on  $X$  with the slope  $b$ .

(a) Show that 
$$R^2 = b^2 \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{\sum_{i=1}^n (Y_i - \bar{Y})^2}$$

- (b) Prove that the coefficient of determination is equal to the square of the sample correlation between  $X$  and  $Y$ .
- (c) Let  $b^*$  be the slope of the least square regression of  $X$  on  $Y$ , and  $r$  the sample correlation between  $X$  and  $Y$ .  
Prove that  $bb^* = r^2$ .

4. (21 points) The data gathered for a two-way factorial design follow.

		Treatment 1		
		A1	A2	A3
Treatment 2	B1	23	21	20
	B2	27	24	26
		28	27	27

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Use the two-way ANOVA to analyze these data. Let the significance level  $\alpha = 0.01$ .

- Test whether or not row means all are equal.
- Test whether or not column means all are equal.
- Test whether or not the interaction effects are zero.

(Hint:  $F_{0.01,1,6} = 13.75$ ,  $F_{0.01,2,6} = 10.92$ ,  $F_{0.01,3,6} = 9.78$ ,

$F_{0.01,1,12} = 9.33$ ,  $F_{0.01,2,12} = 6.93$ ,  $F_{0.01,3,12} = 5.95$ .)

5. (16 points) Prove that if  $S^2 (= \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2)$  is the variance of a random sample from an infinite population with the finite variance  $\sigma^2$ , then  $S^2$  is an unbiased estimator of the population variance  $\sigma^2$ ; that is  $E(S^2) = \sigma^2$ .