考試科目	計算机數學(十)	所別	資訊科學	考試時間	月 日 星期 上下午第 節

計算機數學(離散數學部份) (此部份共計 60 分)

I. 選擇或填充 (不倒扣，每題三分)

1. Let $\wedge$, $\vee$, $\rightarrow$ and ~ denote the logical AND, OR, implication and NOT operations, respectively. Then which of the following sentences is *not* a tautology ?
   (a) $(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$
   (b) $p \vee (q \wedge r) \rightarrow (p \vee q) \wedge r$
   (c) $\sim(p \rightarrow q) \rightarrow \sim q$
   (d) $p \wedge (q \vee r) \rightarrow (p \wedge q) \vee r$
2. Let A and B be two regular languages over $\{0,1\}$. Then which of the following languages is not a regular language ?
   (a) $\{ x \in \{0,1\}^* \mid$ the last 3rd bit of x is "1" $\}$
   (b) $\{ x \in \{0,1\}^* \mid$ There is a bit string y with $|y| = |x|$ and $xy \in A \}$
   (c) $\{xx \mid x \in \{0,1\}^* \}$
   (d) $\{xy \mid x \in A$ and $y \notin B \}$
3. Which of the following statements about tree is FALSE ?
   (a) There exists a tree with degrees 3,3,2,2,1,1,1,1.
   (b) Every tree is bipartite.
   (c) Every tree is planar.
   (d) If two trees have the same number of vertices and and the same degrees, then the two trees are isomorphic.
4. Which of the following statements is TRUE?
   (a) There exists a connected planar simple graph with 7 vertices, 9 edges and 5 regions.
   (b) For all connected multi-graphs G, if the vertices of G have degrees 2,2,2,3,4,4, respectively, then it has an Eular circuit.
   (c) The chromatic numbers of all planar graphs are less than 4.
   (d) There exits an irreflexive and transitive relation which is not antiymmetric.

Let $A = \{(x,y) \mid x, y \in \{1,2,3,4,6\}\}$ and define a relation R on A with (x_1,y_1) R (x_2,y_2) if and only if $x_1 y_2 = x_2 y_1$. It is easy to show that R is an equivalence relation on A. Now answer the following two questions:

5. What is the size of the equivalence class [(2,4)] ?
   (a) 2　　(b) 3　　(c) 4　　(d) 5

備考	試題隨卷繳交

考試科目	計算機數學(一)	所別	資訊科學	考試時間	月　日 星期　上下午第　節

6. How many equivalence classes are there induced by the R relation ?
   (a) 11　(b) 13　(c) 17　(d) 25
7. Let $S_0, S_1, S_2, \ldots$ be a sequence defined as follows: $S_0 = 8$, $S_1 = 10$, and $S_K = 5S_{k-1} - 6S_{k-2}$ for all $k \geq 2$. Let the solution of S_n be of the form: $S_n = a\,2^n + b\,3^n$ for all $n \geq 0$. Then what is the value of a + b ?
   (a) 6　(b) 8　(c) -6　(d) -4
8. What is the maximal number of leaves for a rooted 3-ary tree of height $n \geq 0$. Note the height of a tree with only a single node is 0.
9. What is the sum of all coefficients of all terms in the expansion of the $(3x - y)^{20}$?
10. Find a positive integer $n < 231$ such that $n \equiv 2 \pmod 3$, $n \equiv 5 \pmod 7$ and $n \equiv 2 \pmod{11}$.

II 計算與證明（共30分）

11. (10 pts) For all integer $n > 0$, define $H(n) = 1 + 1/2 + 1/3 + \ldots + 1/n$.
   (a) Show that for all integer $n \geq 0$, $(n + 2)/2 \leq H(2^n) \leq n + 2$.
   (b) Using the above result to show that $H(n) = \Theta(\log n)$.

12. (20pts) A tournament is a directed graph without self-loop such that if $u \neq v$ are two vertices of the graph, then exactly one of (u,v) and (v,u) is an edge of the graph.
   (a) How many different tournaments are there in a set of $n > 0$ vertices ? (5 pts)
   (b) What is the sum of the in-degrees and out-degrees of all vertices in a tournament with $n > 0$ vertices? (5pts)
   (c) Show that every tournament has at least a Hamilton path ? (Note: A Hamilton path is a path passing through all vertices exactly once. This theorem can be proved by induction) (10 pts)

備考	試題隨卷繳交

考試科目	計算機數學(一) 機率	所別	資科系	考試時間	月 日上下午第 節 星期

四、(1) Three fair dice are rolled. You are allowed to bet 1 dollar on the occurrence of one of the integers 1, 2, 3, 4, 5, 6. Suppose you bet on the occurrence of a 5. Then if one 5 occurs (on the 3 dice) you win 1 dollar, if two 5's occur you win 2 dollars, and if three 5's occur you win 3 dollars. If no 5's occur you lose 1 dollar. Let V be the net amount you win in one play of this game. What is the probability density (or mass) function of V? What are its mean and variance? Is this a fair game? (10%)

(2) Suppose that we are going to drive a car to an athletic event. From past experience in making this trip we are willing to assume that our driving time is equally likely to be anywhere from 20 to 30 minutes. If we let X be the number of minutes it will take us to get there, what is the probability density function of X? What is the cumulative distribution function of X? What are the mean and variance of X? (10%)

(3) At a certain manufacturing plant, accidents have been occurring at the rate of 1 every 2 months. Assuming that the accidents occur independently, what is the expected number of accidents per year? What is the standard deviation of the number of accidents per year? What is the probability of there being no accidents in a given month? (10%)

(4) Students at a large university are required to take an entrance exam when entering the school (scored on a basis of 100 points possible). Let X be the score made by a particular incoming student (who does go on to graduate) and let Y be his graduating quality point ratio (4 points = A). The joint density function of X and Y is observed to be

$$f_{X,Y}(x,y)=\begin{cases}\frac{1}{50}, & \text{for } 2<y<4,\ 25(y-1)<x<25y\\ 0, & \text{otherwise}\end{cases}$$

Find μ_X, μ_Y, and ρ_{XY}. Explain the significance of ρ_{XY}. (10%)

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命題委員： （簽章） 90 年 4 月 13 日