考試科目	計算机數學(二)	所別	資訊科學	考試時間	4月22日上午第4節 星期日

計算機數學(離散數學部份) (此部份共計 60 分)

I. 選擇或填充 (不倒扣，每題三分)

1. Let $\wedge$, $\vee$, $\rightarrow$ and ~ denote the logical AND, OR, implication and NOT operations, respectively. Then which of the following sentences is *not* a tautology ?
   (a) $(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$
   (b) $p \vee (q \wedge r) \rightarrow (p \vee q) \wedge r$
   (c) $\sim(p \rightarrow q) \rightarrow \sim q$
   (d) $p \wedge (q \vee r) \rightarrow (p \wedge q) \vee r$
2. Let A and B be two regular languages over $\{0,1\}$. Then which of the following languages is not a regular language ?
   (a) $\{ x \in \{0,1\}^* \mid$ the last 3rd bit of x is "1" $\}$
   (b) $\{ x \in \{0,1\}^* \mid$ There is a bit string y with $|y| = |x|$ and $xy \in A \}$
   (c) $\{xx \mid x \in \{0,1\}^* \}$
   (d) $\{xy \mid x \in A$ and $y \notin B \}$
3. Which of the following statements about tree is FALSE ?
   (a) There exists a tree with degrees 3,3,2,2,1,1,1,1.
   (b) Every tree is bipartite.
   (c) Every tree is planar.
   (d) If two trees have the same number of vertices and and the same degrees, then the two trees are isomorphic.
4. Which of the following statements is TRUE?
   (a) There exists a connected planar simple graph with 7 vertices, 9 edges and 5 regions.
   (b) For all connected multi-graphs G, if the vertices of G have degrees 2,2,2,3,4,4, respectively, then it has an Eular circuit.
   (c) The chromatic numbers of all planar graphs are less than 4.
   (d) There exits an irreflexive and transitive relation which is not antiymmetric.

Let $A = \{(x,y) \mid x, y \in \{1,2,3,4,6\}\}$ and define a relation R on A with (x_1,y_1) R (x_2,y_2) if and only if $x_1 y_2 = x_2 y_1$. It is easy to show that R is an equivalence relation on A. Now answer the following two questions:

5. What is the size of the equivalence class $[(2,4)]$?
   (a) 2 (b) 3 (c) 4 (d) 5

備考	試題隨卷繳交

命題委員： （簽章） 90 年 4 月 10 日

考試科目	計算机數學(一)	所別	資訊科學	考試時間	月 日 星期 上下午第 節

6. How many equivalence classes are there induced by the R relation ?
   (a) 11　(b) 13　(c) 17　(d) 25
7. Let $S_0, S_1, S_2, \ldots$ be a sequence defined as follows: $S_0 = 8$, $S_1 = 10$, and $S_K = 5S_{k-1} - 6S_{k-2}$ for all $k \geq 2$. Let the solution of S_n be of the form: $S_n = a\, 2^n + b\, 3^n$ for all $n \geq 0$. Then what is the value of $a + b$?
   (a) 6　(b) 8　(c) -6　(d) -4
8. What is the maximal number of leaves for a rooted 3-ary tree of height $n \geq 0$. Note the height of a tree with only a single node is 0.
9. What is the sum of all coefficients of all terms in the expansion of the $(3x - y)^{20}$?
10. Find a positive integer $n < 231$ such that $n \equiv 2 \pmod 3$, $n \equiv 5 \pmod 7$ and $n \equiv 2 \pmod{11}$.

II 計算與證明（共 30 分）

11. (10 pts) For all integer $n > 0$, define $H(n) = 1 + 1/2 + 1/3 + \ldots + 1/n$.
   (a) Show that for all integer $n \geq 0$, $(n + 2)/2 \leq H(2^n) \leq n + 2$.
   (b) Using the above result to show that $H(n) = \Theta(\log n)$.

12. (20pts) A tournament is a directed graph without self-loop such that if $u \neq v$ are two vertices of the graph, then exactly one of (u,v) and (v,u) is an edge of the graph.
   (a) How many different tournaments are there in a set of $n > 0$ vertices ? (5 pts)
   (b) What is the sum of the in-degrees and out-degrees of all vertices in a tournament with $n > 0$ vertices? (5pts)
   (c) Show that every tournament has at least a Hamilton path ? (Note: A Hamilton path is a path passing through all vertices exactly once. This theorem can be proved by induction) (10 pts)

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命題委員：　（簽章）　90 年 4 月 10 日

考試科目	計算機數學(二) 線性代數	所別	資科系	考試時間	月 日 上/下 午第 節 星期

亚 (1) True or false: a brief proof or counterexample is needed. (24%)

(a) Every invertible matrix can be written as a product of elementary matrices.

(b) Similar matrices have the same eigenvalues and eigenvectors.

(c) If W is a subspace of a vector space of V, then every basis of W can be expanded into a basis for V. Conversely, every basis of V can be reduced to a basis of W.

(2) Find all solutions to the following linear system by reducing the associated matrix to row echelon form.

$$\begin{aligned} 2x - y - 3z + w &= 2 \\ x - 2z + w &= 1 \\ -3x + y + z + 2w &= 3 \end{aligned} \qquad (8\%)$$

(3) Let V be the space of all 2x2 matrices with real entries. Let T: V→V be defined by T(A) = A' where A' is the transpose matrix of A. Show that T is a linear transformation. Is T diagonalizable? Explain your answer. (8%)

備考	試題隨卷繳交

305

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