考試科目	數學	所別	經研所	考試時間	月 日 星期 上午/下午 第 節

1. (10%) Find possible local extreme points for the function
$$F(x,y)=\int_0^x (t\sin t - x - y\cos t)^2 dt$$

2. (10%) Use Cramer's rule to compute the solution of the following system:
$$2x_1 + x_2 = 7$$
$$-3x_1 + x_3 = -8$$
$$x_2 + 2x_3 = -3$$

3. (15%) Let f be defined by $f(x,y)=x^2y^3-(y+1)^2$
(a) Find all critical points for f and classify them.
(b) Compute the double integral
$$I=\iint_R f(x,y)dydx, \quad R=[0,2]\times[-2,1]$$

4. (15%)
(a) Calculate the directional derivative of
$$f(x,y,z)=xy\ln(x^2+y^2+z^2)$$
at the point (1,1,1) in the direction determined by the vector from the point (3,2,1) to the point (-1,1,2).
(b) Determine the direction of steepest increase for f at (1,1,1).

5. (15%) Determine true or false, and justify your answer.
(a) A square matrix is singular if and only if 0 is an eigenvalue of A.

(b) Every Markov matrix has an eigenvalue of 1.

(c) The latent roots of a real symmetric matrix are all real.

6. (15%) Consider the matrix
$$A=\begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$$
Find P such that $P^{-1}AP=\begin{bmatrix} \lambda & 1 \\ 0 & \lambda \end{bmatrix}$, where λ is the only eigenvalue of A.

7. (20%) Solve the following linear differential equation system, draw the phase diagram, and find the equations for the saddle path and the monotonically explosive path, respectively.
$$\dot{y}_1 = 2y_1 - 9y_2$$
$$\dot{y}_2 = -3y_1 - 4y_2$$

備考	試題隨卷繳交

命題委員： （簽章） 91 年 12 月 9 日