1. Let $f(x) = \begin{cases} x^2 & \text{if } x \le 2 \\ mx + b & \text{if } x > 2 \end{cases}$

Find the values of m and b that make f differentiable every where.　　(10 points)

2. Use Newton's method to find $\sqrt[6]{2}$ correct to four decimal places.　　(10 points)

3. If $F(x) = \int_1^x f(t)\,dt$, where $f(t) = \int_1^{t^2} \frac{\sqrt{1+u^4}}{u}\,du$, find $F''(2)$.　　(10 points)

4. Evaluate the integral $\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}$.　　(10 points)

5. Phenomena such as waiting times and equipment failure times are commonly modeled by exponentially decreasing probability density functions of the form

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ ke^{-ct} & \text{if } x \ge 0 \end{cases}, \text{ where } k \text{ and } c \text{ are positive constants.}$$

(a) For $f(x)$ being a probability density function, what is the value of k?　　(5 points)
(b) Find the mean of the exponential distribution.　　(5 points)
(c) Suppose the average waiting time for a customer's call to be answered by a company is five minutes. Find the possibility that a customer waits more than five minutes to be answered.　　(5 points)

6. Investigate the sequence $\{a_n\}$ defined by the recurrence relation

$$a_1 = 2 \qquad a_{n+1} = \frac{a_n + 6}{2} \quad \text{for } n = 1, 2, 3, \ldots$$

(a) Apply the Monotonic Sequence Theorem to show that $\lim_{n \to \infty} a_n$ exists. [Hints: Show that the sequence $\{a_n\}$ is monotonic and bounded].　　(10 points)
(b) Find $\lim_{n \to \infty} a_n$.　　(5 points)

7. Find the local maximum and minimum values and saddle points of
$$f(x, y) = x^4 + y^4 - 4xy + 1.$$　　(15 points)

8. (Pareto's Law) According to the economist Vilfredo Pareto (1848-1923), the rate of decrease of the number of people y in a stable economy having an income of at least x dollars is directly proportional to the number of such people and inversely to their income x. That is modeled by the differential equation
$$\frac{dy}{dx} = -k\frac{y}{x}.$$

(a) Solve this differential equation.　　(5 points)
(b) In 1995, 7.1 million people in the United State earned over $75,000 and 64.7 million people earned over $25,000. Assume that Pareto's Law holds and use the result derived in (a) to sketch the graph that describes the relation of x and y.　　(5 points)
(c) Determine the number of people (in millions) who earn over $20,000.　　(5 points)
[Data Source: U.S. Bureau of Census]