

考試科目	計算機數學(二)	所別	資訊科學	考試時間	4月21日 上午 9:00 至 11:30 節
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I 選擇與填充 (35 %; 不倒扣)

- (4%) Let $f(n)$ be an increasing function satisfying the recurrence relation: $f(n) = 7f(n/2) + 15n^2/4$. Then which of the following statements are correct? (多選)
 - $f(n) = O(n^2)$
 - $f(n) = o(n^2)$
 - $f(n) = O(n^3)$
 - $f(n) = o(n^3)$
 - $f(n) = \Theta(n^3)$
- (4%) Which of the following statement is *not* valid in Boolean algebras.
 - $x(y+z) = xy + xz$
 - $x(x+y) = x$
 - $(x+y)(x+z) = x + yz$
 - $x(\sim y) + (\sim x)y = x+y$
- (4%) What is the proposition a proof of which *does not* imply the validity of the implication $p \rightarrow q$?
 - $p \rightarrow \text{false}$
 - $\sim p \rightarrow \sim q$
 - $\sim p \text{ or } q$
 - $\sim q \rightarrow \text{false}$
 - $\sim(p \text{ and } \sim q)$
- (4%) What is the number of non-negative integer solutions to the equation $x + y + z < 18$ with $x \geq 1$, $y \geq 2$ and $z \geq 3$.
- (4%) What is the probability of winning the 3 out-of-6 prize in playing Taiwan lottery for correctly choosing 3 (but not 4, 5 or 6) numbers out of six integers chosen between 1 and 42, inclusively, by a fair random process.
- (5%) What is the least positive integer x satisfying the system of congruences : $x \equiv 2 \pmod{5}$, $x \equiv 5 \pmod{11}$ and $x \equiv 11 \pmod{17}$.
- (10%) Which of the following statements are correct? (多選; 每答對一小題給 2%)
 - All spanning trees of a graph have the same number of edges.
 - If $G = (V, E)$ is a multigraph containing no isolated or pending vertices, then the number of edges $|E|$ of G is always less than or equal to the number of vertices $|V|$ of G .
 - The chromatic number of every graph is less than or equal to 4.
 - There exists a Euler circuit for every complete simple graph K_5 with 5 vertices.
 - There exists one and only one path between every two vertices of a tree.

考試科目	計算機數學(二)	所別	資訊科學	考試時間	4月21日 上午 第二節 星期日 (下)
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II 計算與證明 (25 %)

8. (10 %) Let G be a directed graph with n vertices, x and y two distinct vertices of G . Show that if there is a path from x to y then there must exist a path of length less than n from x to y .
9. (15%) Let \leq be a preorder on a nonempty set S (i.e., \leq is a reflexive and transitive relation on S). Define a new relation \equiv on S as follows : for all $x, y \in S$, $x \equiv y$ iff $x \leq y$ and $y \leq x$.
- (a) Show that \equiv is an equivalence relation on S . [6 %]
- Let \ll be a relation on the set S/\equiv of all equivalence classes of S such that for all A and B belonging to S/\equiv , $A \ll B$ iff there exist $x \in A$ and $y \in B$ with $x \leq y$.
- (b) Show that the relation \ll is a partial order on S/\equiv . [9 %]

考試科目	計算機數學(中)所別	資料系	考試時間	4月21日 星期日 下午第二節
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IV.

$$\text{設 } A = \begin{bmatrix} 1 & 1 & -2 & 0 & -1 \\ 2 & 2 & -1 & 0 & 1 \\ 1 & 1 & -2 & 3 & -1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

(八分) (1) 將 A 化為簡約列梯形 (reduced row-echelon form). (必須列出過程)

(八分) (2) 求齊次系統 (homogeneous system) $Ax = 0$ 的解空間 (solution space) 之基底 (basis),

其中 $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$, 0 是零向量.

(八分) (3) A 的秩 (rank) 是多少? (必須說明理由)

(八分) IV (i) R^3 中的三個向量 $v_1 = (3, 0, -4)$, $v_2 = (5, -1, 2)$, $v_3 = (1, 1, 3)$ 是否線性獨立 (linearly independent)? 並將 $v = (16, 1, 9)$ 表示成 v_1, v_2, v_3 的線性組合.

(八分) (2) $W = \left\{ \begin{bmatrix} a & c \\ c & b \end{bmatrix} \mid a, b, c \text{ 是實數} \right\}$ 是不是

$V = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mid a, b, c, d \text{ 是實數} \right\}$ 的子空間?

(必須說明理由). 如果是, 也求出 W 的基底與維數 (dimension).