

考試科目	數學	所別	經濟所(甲組) 261	考試時間	4月20日(上) 星期日下午第2節
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1. (10%) Indicate in a drawing the convex hull of the set  $S = \{(x, y) : x^2 + y^2 \leq 1 \text{ and } y^2 \leq x\}$  and find the extreme points (if any) of the convex hull.

2. (10%) A convex function can not be a quasiconcave function. True or false? Reason.

3. (10%) Suppose a square matrix has 4 latent roots: 1, 3, -5 and -4. Could you determine its determinant and its trace?

4. (10%) For  $x \in R^3$  let

$$Q(x_1, x_2, x_3) = 6x_1^2 + 25x_2^2 + 9x_3^2 - 60x_2x_3 + 40x_1x_3 - 6x_1x_2$$

(a) Write this quadratic form as  $x^T Ax$ .

(b) Examine the quadratic form are positive definite, negative definite or indefinite.

5. (10%) Let  $f(x, y) = e^x \ln(1 + y)$ . Approximate  $f(x, y)$  by a second-order polynomial in  $x$  and  $y$  at point  $(0, 0)$ .

6. (10%) Use the Gauss-Jordan elimination method to decide whether the following matrix has an inverse

$$A = \begin{bmatrix} 12 & 13 & 14 \\ 15 & 16 & 17 \\ 18 & 19 & 20 \end{bmatrix}$$

7. (20%)

(a) Compute  $I = \iint_A (x^2 + y^2 - 1) dx dy$ , where  $A$  is the domain in the  $xy$ -plane bounded by  $x + y = 1$ ,  $x + y = 5$ ,  $x - y = -1$  and  $x - y = 1$ .

(b) Assume transform the  $xy$ -plane to  $uv$ -plane by  $u = x - y$ , and  $v = x + y$ . Re-compute  $I$  in the  $uv$ -plane.

8. (20%) Suppose there are only two firms in a homogeneous good market. Let  $x_t$  and  $y_t$  be the two firms' output levels in period  $t$ , respectively. The market demand curve is  $p(x + y) = 120 - (x + y)$ .

Assume each firm makes a Cournot assumption that each firm chooses its output for  $t + 1$  to a maximize profit in belief that the other firm will maintain its output level at the period  $t$  value. Assume costs are zero. Determine the market equilibrium price and output level, and each firm's profit. Is the equilibrium stable?