考試科目	微積分	系所別	財務金融研究所	考試時間	2月 5日(六) 第2節

Problem 1 (10 points)
Evaluate the iterated integral

$$\int_0^a \int_x^a sin(y^2)dydx,\ a > 0.$$

Problem 2 (15 points)

(a) Evaluate

$$\lim_{x\to 0} \frac{1 - cos\ x}{x^2}.$$

(b) Evaluate

$$\lim_{n\to\infty} \frac{1 - cos(\frac{1}{n})}{1 - cos(\frac{1}{n+1})}.$$

(c) Find the interval of convergence of the power series

$$\sum_{n=1}^{\infty}(1 - cos(\frac{1}{n}))x^n.$$

Problem 3 (10 points)
Use the method of Lagrange multiples to find the extreme values of z on the curve of intersection of $x^2 + z^2 = 1$ and $y^2 + z^2 + z = 1$.

Problem 4 (10 points)
Determine whether the series converges absolutely, or converges conditionally, or diverges.

(a)

$$\sum_{n=1}^{\infty}(-1)^n \frac{3 \cdot 5 \cdot 7 \cdots (2n+1)}{n! \cdot 3^n}.$$

(b)

$$\sum_{n=1}^{\infty} ln(1 + \frac{1}{\sqrt{n}}).$$

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考試科目	微積分	系所別	風險管理與保險學系精算科學組	考試時間	2月5日(五) 第2節

Problem 5 (10 points)
Evaluate the integral

$$\int \frac{dt}{t - \sqrt{1 - t^2}}.$$

Problem 6 (10 points)
Evaluate the improper integral $\int_0^\infty x^n e^{-x} dx$, n is a positive number.

Problem 7 (15 points)
Consider the cardioid given by $r = 1 - cos\ \theta$, $0 \le \theta \le 2\pi$.

(a) Find the area enclosed by this curve.

(b) Find the length of this curve.

Problem 8 (20 points)
Evaluate the limits.

(a)

$$\lim_{x \to \infty} (1 + \frac{3}{x} + \frac{5}{x^2})^x.$$

(b)

$$\lim_{x \to 0} \frac{\int_{cos\ x}^{1} \frac{2}{t} dt - x^2}{x^4}.$$

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。