考試科目	數理統計學	系所別	統計學系	考試時間	2 月 5 日(五) 第二節

1. (35pt)**此大題請直接填寫答案，不需要提供過程說明**

a. (5pt) Let X be one observation from a $N(0,\theta)$ distribution. Find the sufficient statistic for θ.

b. (5pt) Let $X_1, X_2, \ldots, X_n$ be i.i.d. from the density $f(x;\theta) = \frac{1}{\pi} \frac{1}{1+(x-\theta)^2}$. Find the sufficient statistic for θ.

c. (5pt) Let X be one observation from the density

$$f(x;\theta) = \left(\frac{\theta}{2}\right)^{|x|} (1-\theta)^{1-|x|}, x = -1, 0, 1;\ \ 0 \le \theta \le 1.$$

Find an unbiased estimate for θ.

d. (10pt) Let $X_1, X_2, \ldots, X_8$ be a random sample from Uniform$(\mu - \sqrt{3}\sigma,\ \mu + \sqrt{3}\sigma)$, where μ and $\sigma > 0$ are the unknown parameters. Find the MLE of μ and σ based on the 8 observations:

$$X_1 = 2.88,\ X_2 = -4.04,\ X_3 = -3.26,\ X_4 = -3.57,$$

$$X_5 = -0.09,\ X_6 = 0.25,\ X_7 = -2.58,\ X_8 = 4.59.$$

e. (5pt) Let X be one observation from Uniform$(\theta,\ \theta+1)$. For testing H_0: $\theta = 0$ vs. H_1: $\theta > 0$, find the power function for the decision rule that we reject H_0 if $X > 0.9$

f. (5pt) Let $X_1, X_2, \ldots, X_n$ be a random sample from $P(X=k) = \theta(1-\theta)^{k-1}, k = 1,2, \ldots$. Find the MLE of θ.

2. (20pt) Let the distribution of X be Uniform(0, 1). Conditional on $X = x$, let the distribution of Y be the normal distribution with mean x and variance x^2.

a. (8pt) Find the distribution of Y.

b. (12pt) Are $\frac{Y}{X}$ and X independent? Prove or disprove it with your work.

3. (15pt) Let $X_1, X_2, \ldots, X_n$ be a random sample from the density $f(x;\theta) = \frac{1}{\theta}\exp\left(\frac{(1-\theta)}{\theta}\ln x\right), 0 < x < 1.$

Test H_0: $\theta \le \theta_0$ vs. H_1: $\theta > \theta_0$. Find a uniformly most powerful size α test if such exists.

4. (30pt) Let $X_1, X_2, \ldots, X_n$ be a random sample from the density

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$$f(x;\ \theta)=\begin{cases}\frac{2x}{\theta},\ 0<x\le\theta\\ \frac{2(1-x)}{1-\theta},\ \theta<x\le 1\end{cases},\ \text{where }\ 0\le\theta\le 1.$$

a. (5pt) Find $E(X_1)$

b. (4pt) Find the moment estimate of θ

c. (7pt) Find the maximum likelihood estimate of θ for $n=1$

d. (7pt) For $n=1$ find a complete sufficient statistic if such exists.

e. (7pt) Find a UMVUE of θ for $n=1$ if such exists

備 註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。