

考試科目	微積分	系所別	國貿系	考試時間	2月5日(五)第4節
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※Show all your work. Unjustified answers will receive no credit.

1. Evaluate the following integrals:

(a)  $\int_1^e \ln \sqrt[3]{x} dx$

(b)  $\int_0^1 e^{(x-e^x)} dx$

(c)  $\int_0^1 x^2 e^x dx$

(d)  $\int_0^2 \frac{10x^2-12}{x^4-5x^2-36} dx$  (20%)

2. Evaluate the following limits:

(a)  $\lim_{x \rightarrow 1} \frac{x^{2x}-1}{x-1}$  (b)  $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{\pi i}{n^2} \sin\left(\frac{\pi i^2}{n^2}\right)$  (10%)

3. For the demand equation  $x^2 + 4xp + 4p = 18000$ , find  $\frac{dp}{dx}$  at  $p = 11$  and  $x = 2$ . (10%)

4. Use Lagrange multipliers (拉氏乘子法) to find the points on the sphere  $x^2 + y^2 + z^2 - 2y = 35$  closest to and farthest from the point  $(1, -1, 2)$ . (10%)

5. Find the area between the graphs  $y = 2^x$  and  $y = 2^{-x}$  over  $-1 \leq x \leq 2$ . (10%)

6. Determine convergence or divergence of the series  $\sum_{n=0}^{\infty} (\sqrt{n^2+1} - n)$ . (10%)

7. Let  $f(x) = \sqrt[3]{x+1}$ . Use the definition of derivative (no differentiation rule) to find  $f'(x)$ . (10%)

8. Use total differential (全微分) to approximate the value of  $(1.01)^7 \times (1.98)^4$ . (10%)

9. Evaluate the iterated integral  $\int_0^4 \int_{\sqrt{y}}^{\sqrt{8-y}} \frac{1}{(1+y)^2} dx dy$ . (10%)

備

註

- 一、作答於試題上者，不予計分。  
二、試題請隨卷繳交。