考試科目	統計學	系所別	經濟學系	考試時間	2月4日(四) 第四節

注意事項:
(1) 請依題號順序作答。
(2) 不可使用計算機。
(3) 答題若過程錯誤 (或沒有過程) 但答案正確, 將以「零分」計算。

1. (10%) Let X be a random variable with probability density function given by

$$f(x) = \begin{cases} ke^{x}, & x < 0, \\ 0 & \text{otherwise.} \end{cases}$$

Find the moment generating function for X.

2. (Total 20%) Let Y_1 and Y_2 denote the jointly continuous random variables with joint density function $f_{Y_1,Y_2}(y_1, y_2)$, where $-\infty < y_1, y_2 < \infty$. Please show

(1) (10%) $\mathbb{E}\big[\mathbb{E}[Y_2|Y_1]\big] = ?$

(2) (10%) $\text{Var}(Y_2|Y_1 = y_1) = ?$

3. (Total 30%) Let Y denote the length of life (in hundreds of hours) of electronic components. These components frequently fail immediately upon insertion into a system. It has been observed that the probability of immediate failure is 1/3. If a component does not fail immediately, the distribution for its length of life has the exponential density function

$$f(y) = \begin{cases} e^{-y}, & y > 0, \\ 0, & \text{otherwise.} \end{cases}$$

(1) (10%) Find the cumulative distribution function for Y.

(2) (10%) Evaluate $\mathbf{P}(Y > 10)$.

(3) (10%) Find the mean and variance of Y.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

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4. (Total 40%) Consider a multiple linear regression model as

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_k X_k + u,$$

where β_0 is the intercept, β_j is the slope parameter associated with X_j ($j = 1, 2, \ldots, k$). Given n observations on $Y, X_1, \cdots, X_k$, $\{(y_i, x_{i1}, x_{i2}, \cdots, x_{ik}) : i = 1, 2, \ldots, n\}$. Let $\widehat{\beta}_j$, $j = 0, 1, \ldots, k$, be the ordinary least squares (OLS) estimates for the parameters of this model, $\widehat{u}_i$ denote the residual associated with the $i-$th observation, and t_j denote the t-ratio for β_j $j = 0, 1, \ldots, k$,.

(1) (10%) Find $\sum_{i=1}^{n} \widehat{u}_i$ and $\sum_{i=1}^{n} \widehat{u}_i x_{ik}$.

(2) (10%) Under the classical linear model assumptions, please show how to construct the F test statistic for the joint test $H_o : \beta_1 = \beta_2, \beta_3 = 0$, and write down the decision rule given the significance level α.

(3) For $i = 1, \ldots, n$, let $y_i^* = (y_i - \bar{y})/\hat{\sigma}_y$, $x_{ij}^* = (x_{ij} - \bar{x}_j)/\hat{\sigma}_j$, $j = 1, 2, \ldots, k$, be the standardized version of the data, where $\bar{y}$ and $\hat{\sigma}_y$ are the sample mean and sample standard deviation for y over these n observations, and $\bar{x}_j$ and $\hat{\sigma}_j$ are the sample mean and sample standard deviation for x_j, $j = 1, 2, \ldots, k$. Now we regress y^* on 1 and x_j, $j = 1, 2, \ldots, k$, to yield the estimated model as

$$y_i^* = \widehat{b}_0 + \widehat{b}_1 x_{i1}^* + \widehat{b}_2 x_{i2}^* + \cdots + \widehat{b}_k x_{ik}^* + \hat{v}_i, \quad i = 1, 2, \ldots, n,$$

where $\widehat{b}_j$, $j = 0, 1, \ldots, k$, are the associated OLS estimates.

a. (10%) Find the OLS estimates $\widehat{b}_j$, $j = 0, 1, \ldots, k$ in terms of $\widehat{\beta}_j$, $j = 0, 1, \ldots, k$.

b. (10%) Find the t-ratio for b_j in terms of t_j, $j = 0, 1, \ldots, k$.

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