國立政治大學 110 學年度碩士班暨碩士在職專班招生考試試題

考 試 科 目	計算機數學	系 所 別	資訊科學系	考 試 時 間	2 月 5 日(五) 第 4 節

I. 離散數學: 共 19 題(1-17，26-27)，1-17 為選擇題，26-27 為簡答題，共 60%

II. 線性代數: 共 9 題(18-25，28)，18-25 為選擇題，28 為簡答題，共 40%

非選擇題請書寫必要的解題過程，僅書寫答案而缺乏必要的過程，無法獲得該題滿分。可使用中文或英文作答，力求書寫工整，如字跡潦草，無法閱讀，將影響評分。

選擇題請在答案卡上作答，否則不予計分。

1. (3%) Let ϕ denote an empty set. Which of the following statements is true?
(A) $\phi \in \phi$ (B) $\phi \subset \phi$ (C) $\phi \subset \{\phi\}$ (D) $|\phi| = 1$

2. (3%) Let A, B, C be finite sets. Which of the following statements is false?
(A) $|A \cup B| = |A| + |B| - |A \cap B|$ (B) $P(A \cup B) = P(A) \cup P(B)$
(C) $A - (B \cup C) = (A - B) - C$ (D) $A \cap (B \oplus C) = (A \cap B) \oplus (A \cap C)$

3. (3%) Let A, B, C be finite sets. Which of the following statements is true?
(A) If $A \cup C = B \cup C$, then $A = B$ (B) If $A \cap C = B \cap C$, then $A = B$
(C) If $A - C = B - C$, then $A = B$ (D) If $A \oplus C = B \oplus C$, then $A = B$

4. (3%) Let $p = 101$ be a prime and $a = 2$ be a primitive root modulo p. Please determine the value of $(a^7)^9 \bmod p$.
(A) 88 (B) 89 (C) 90 (D) 91

5. (3%) Let a, b, k, n be integers. Which of the following statements is not correct?
(A) $a \equiv b \pmod{n} \Rightarrow ka \equiv kb \pmod{n}$
(B) $a \equiv b \pmod{kn} \Rightarrow a \equiv b \pmod{n}$
(C) $a \equiv b \pmod{n} \Rightarrow ka \equiv kb \pmod{kn}$
(D) $a \equiv b \pmod{n} \Rightarrow a \equiv b \pmod{kn}$

6. (3%) Which of the following statements is true?
(A) $p \to q \equiv \neg(p \vee q)$ (B) $\neg(p \leftrightarrow q) \equiv \neg p \leftrightarrow \neg q$
(C) $\neg\exists x\ (Q(x) \wedge R(x)) \equiv \forall x(\neg Q(x) \vee R(x))$ (D) $(p \wedge (p \to q) \to q) \equiv T$

7. (3%) Which of the following statements is false?
(A) $\forall x[Q(x) \wedge R(x)] \to \forall x\ Q[x] \wedge \forall x\ R(x)$ (B) $\forall x[Q(x) \vee R(x)] \to \forall x\ Q(x) \vee \forall x\ R(x)$
(C) $\exists x[Q(x) \wedge R(x)] \to \exists x\ Q[x] \wedge \exists x\ R(x)$ (D) $\exists x[Q(x) \vee R(x)] \to \exists x\ Q(x) \vee \exists x\ R(x)$

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8. (3%) How many bit strings are there containing exactly five 0s and fourteen 1s if every 0 must be immediately followed by two 1s?
(A) 112 (B) 124 (C) 126 (D) 132

9. (3%) Find the number of ways in which nine identical blocks can be given to four children A, B, C, D, where child A gets at most two blocks.
(A) 100 (B) 136 (C) 142 (D) 164

10. (3%) How many ways are there to put five temporary employees into four identical offices?
(A) 48 (B) 51 (C) 56 (D) 60

11. (3%) Let $x = a + bk$ be the solution of the system of congruences $x \equiv 7 \pmod{9}$, $x \equiv 4 \pmod{12}$, and $x \equiv 16 \pmod{21}$, where a and b are the smallest positive integer, $k \in Z$. Which is the correct value of $(2a + b) \bmod 11$?
(A) 9 (B) 7 (C) 4 (D) 6

12. (3%) Let A be a set with n elements. Which of the followings is the number of binary relations on A which are symmetric and antisymmetric, and not irreflexive?

(A) 2^n (B) $2^n - 1$ (C) $(2^n - 1) \cdot 3^{\frac{n^2-n}{2}}$ (D) $(2^n - 2) \cdot 3^{\frac{n^2-n}{2}}$

13. (3%) Let A be a set with n elements. Which of the followings is the number of binary relations on A which are reflexive and not symmetric?

(A) $2^{n^2} - 2^{n^2-n+1}$ (B) $2^{n^2} \cdot 2^{\frac{n^2-n}{2}}$ (C) $2^{n^2-n} - 2^{\frac{n^2-n}{2}}$ (D) $(2^n - 1) \cdot 2^{\frac{n^2-n}{2}}$

14. (3%) Suppose that someone starts a chain letter. Each person who receives the letter is asked to send it to four other people. Let k be the number of people who have seen the letter, including the first person. Suppose no one receives more than one letter and the chain letter ends after there have been 100 people who read it but did not send it out. which is the correct value of $k \bmod 11$?
(A) 0 (B) 3 (C) 4 (D) 8

15. (3%) Let G be a connected bipartite planar simple graph with 12 vertices, and let e be the maximum value of the number of edges in G. Which of the followings is the value of $e \bmod 11$?

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(A) 3 (B) 7 (C) 9 (D) 10

16. (3%) Consider the following string aaabbcdddddddeeeeeffggggghhhhiii. Suppose we at least require k bits to encode the string. Which of the followings is the value of k mod 11?
(A) 1 (B) 6 (C) 5 (D) 10

17. (3%) Suppose the solution of the recurrence relation $a_n = 7a_{n-1} - 16a_{n-2} + 12a_{n-3} + n4^n$ with $a_0 = -2$, $a_1 = 0$, and $a_2 = 5$, is $a_n = a \cdot 2^n + b \cdot n \cdot 2^{n-1} + c \cdot 3^n + (d_2 n^2 + d_1 n + d_0) \cdot 4^n$. Which of the following is the value of $(a + b + c + d_0 + d_1 + d_2) \bmod 11$?
(A) 9 (B) 8 (C) 7 (D) 6

18. (3%) Determine the number of solutions for the linear system $\begin{cases} x + 2y - z = 2 \\ 2x + 5y - 3z = 1 \\ x + 4y - 3z = 3 \end{cases}$.

(A) no solution (B) a unique solution (C) exactly two solutions (D) infinite many solutions

19. (3%) If $\det\begin{bmatrix} b_1 + c_1 & b_2 + c_2 & b_3 + c_3 \\ a_1 + c_1 & a_2 + c_2 & a_3 + c_3 \\ a_1 + b_1 & a_2 + b_2 & a_3 + b_3 \end{bmatrix} = k \cdot \det\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$, what is the value of k?
(A) 2 (B) 3 (C) 4 (D) 6

20. (3%) Which of the following statements is true?
(A) If none of the vectors in the set $S = \{v_1, v_2, v_3\}$ in R_3 is a multiple of one of the other vectors, then S is linearly independent.
(B) If A is a subspace, then its complement is also a subspace.
(C) If a square matrix A has independent columns, so does A^2.
(D) Every vector space has at least two distinct subspaces.

21. (3%) Which of the following statements is a linear transformation?

(A) $L(x, y, z) = (x + y, y^2, 2z)$ (B) $L\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2x - 3y \\ 3y - 2z \\ 2z + 1 \end{bmatrix}$

(C) $L(p(x)) = \begin{bmatrix} \int_0^1 p(x)d(x) \\ p'(x) \end{bmatrix}$，$p(x) \in P_2$ (D) $L(x, y, z) = (4x, \frac{y^2}{z^2})$

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22. (3%) Which of the following statements is true?
   (A) A square matrix with linearly independent column vectors is diagonalizable.
   (B) If A and B are diagonalizable, so is AB.
   (C) If A be diagonalizable and AB = BA, then B is diagonalizable.
   (D) If A is a 3×3 matrix with distinct eigenvalues 0,1,2, then the matrix $(A + I)$ must be invertible.

23. (3%) Which of the following statements is true?
   (A) Let S be a subset of a vector space. Then $S = (S^{\perp})^{\perp}$
   (B) Let V be a subset of R^n, and if V is orthogonal to W, then $V^{\perp}$ is orthogonal to $W^{\perp}$.
   (C) If u and v are vectors in R^n, and if distance from u to v equals the distance from u to $-v$, then u and v are orthogonal.
   (D) If u, v and w are vectors in R^n, and if u is orthogonal to $v + w$, then u is orthogonal to v and w.

24. (3%) Given the following data points $\{(-1,1), (1,3), (2,3)\}$, suppose that the linear function $y = \frac{b}{a} + \frac{d}{c}x$ is the approximation to fit the data in the least squares sense, please find the value of $(a + b + c + d) \bmod 11$.
   (A) 1 (B) 3 (C) 7 (D) 10

25. (3%) Let $S = \text{span}\{(1\ 3\ 1\ 1)^T, (1\ 1\ 1\ 1)^T, (-1\ 5\ 2\ 2)^T\}$ be a subspace of R^4. Suppose $w_1 = (\frac{a_1}{\sqrt{12}}\ \frac{a_2}{\sqrt{12}}\ \frac{a_3}{\sqrt{12}}\ \frac{a_4}{\sqrt{12}})^T$, $w_2 = (\frac{b_1}{2}\ \frac{b_2}{2}\ \frac{b_3}{2}\ \frac{b_4}{2})^T$, $w_3 = (\frac{c_1}{\sqrt{6}}\ \frac{c_2}{\sqrt{6}}\ \frac{c_3}{\sqrt{6}}\ \frac{c_4}{\sqrt{6}})^T$ is an orthonormal basis of S. Please find the value of $(a_1 + a_2 + a_3 + a_4) + (b_1 + b_2 + b_3 + b_4) + (c_1 + c_2 + c_3 + c_4)$?
   (A) 5 (B) 6 (C) 7 (D) 8

26. (4%) Use generating functions to prove Pascal's identity: $C(n, r) = C(n-1, r) + C(n-1, r-1)$ when n and r are positive integers with $r < n$.[Hint: Use the identity $(1 + x)^n = (1 + x)^{n-1} + x(1 + x)^{n-1}$.]

27. (5%) Let (x_i, y_i), $i = 1,2,3,4,5$, be a set of five distinct points with integer coordinates in the xy plane. Show that there is at least one of the midpoints of the segment joining of these points which has integer coordinates.

28. (16%)
Let $T: P_2 \longrightarrow P_2$ be given by

$$T(p(x)) = p(x-1)$$

Consider the two ordered bases $\beta = \{x^2, x, 1\}$ and $\gamma = \{x, x+1, x^2 - 1\}$.

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(a) (6%)Find $[T]_\beta$ and $[T]_\gamma$.

(b) (6%)Find the matrix S such that $[T]_\gamma = S[T]_\beta S^{-1}$.

(c) (4%)If the basis is γ, find the dimension of $\ker(T)$ and the basis of $\ker(T)$.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。