國立政治大學 110 學年度碩士班暨碩士在職專班招生考試試題

考試科目	線性代數	系所別	應用數學系	考試時間	2 月 5 日(五) 第四節

註：1. 計算題必須要有計算過程，否則沒有分數。

2. 證明題若用到其他定理結果，請簡述該定理敘述。

3. 假設 F 是一個 field，若對任意 $n \in N$ 且 $a \in F\backslash\{0\}$ 皆有 $na \neq 0$，稱 F 的 characteristic 是 0。若存在 $p \in N$ 是質數使得對任意的 $a \in F$ 皆有 $pa = 0$，則稱 F 的 characteristic 是 p。

1. (15%) Prove that there exists a linear transformation $T: R^2 \to R^3$ such that $T(1,1) = (1,0,2)$ and $T(2,3) = (1,-1,4)$. What is $T(8,11)$?
2. (15%) Find linear transformations $U, T: F^2 \to F^2$ such that $UT = T_0$ (the zero transformation) but $TU \neq T_0$.
3. (15%) Let $A, B \in M_{n\times n}(F)$ be such that $AB = -BA$. Prove that if n is odd and F is not a field of characteristic two, then A or B is not invertible.
4. (20%) Prove that every invertible matrix is a product of elementary matrices.
5. (20%) For

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

Find an expression for A^n, where n is an arbitrary positive integer.

6. (15%) Let T be a linear operator on an inner product space V, and suppose that

$$\|T(x)\| = \|x\|$$

for all x. Prove that T is one-to-one.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。