1. (10%) According to Hooke's Law, the force $f(x)$ exerted by a spring that has been expanded a distance x is given by $f(x) = -kx$, where k is a positive constant called the **spring constant**. Find a function U such that $f(x) = -dU/dx$. (Such a function U is called a **potential energy function** for the force f. If $U(0)=0$, then $U(x)$ represents the amount of energy stored in the spring when it is expanded a distance x.)

2. (10%) A company has a daily fixed cost of $5000. If the company produces x units daily, then the daily cost in dollars for labor and materials is $3x$. The daily cost of equipment maintenance is $x^2/2500000$. What daily production minimizes the total daily cost per unit of production? (*Hint*: The cost per unit is the total cost $C(x)$ divided by x.)

3. (10%) Let
$$k(x) = \frac{x^2 + 1}{x^2 - 4}$$
Sketch the function, noting all relevant properties of the Graph of k.

4. (10%) Let
$$f(x) = e^x + \frac{1}{4}e^{-x} \quad \text{for} \ \ 0 \le x \le 1$$
Find the length L of the graph of f.

5. (10%) Let $f(x) = 1 + 2x - x^2$ and $g(x) = x^2 - 2x + 1$, and let R be the region between the graphs of f and g on [0,2]. Find the center of gravity of R.

6. (10%) Let
$$f(x) = e^{(x^2)}, g(x) = e^{-x^2} ; [0,1]$$
Find the volume V of the solid obtained by revolving about the y axis the region between the graphs of f and g on the given interval.

7. (10%) Let
$$f(x) = e^x ; a = 2$$
Find the Taylor series of f about a. Do not be concerned with whether the series converges to the given function.

8. (10%) Evaluate the limit.

$$\lim_{x \to 0} \frac{\sin x - \sin(\sin x)}{x^3}$$

9. (10%) Using the fact that $2 \le \sqrt{4+5x^2} \le 3$ *for* $0 \le x \le 1$, find positive numbers a and b such that

$$a \le \int_0^1 x^4 \sqrt{4+5x^2}\,dx \le b$$

10. (10%) A car going 60 feet per second passes a car going 50 feet per second along a straight highway. The distance between the drivers is 10 feet when they are alongside one another. Let θ denote the angle made by one edge of the highway and the line joining the two drivers. How fast is θ changing when $\theta = \pi/3$?

備　考　試 題 隨 卷 繳 交

命 題 委 員 ：　　　　　-75-　　（簽章） 92 年 3 月 25 日